

Tutorial 7 – Technology, Cost minimisation

1. A firm has the production function $f(x_1, x_2) = (\sqrt{x_1} + 3\sqrt{x_2})^2$. The price of factor 1 is w_1 while the price of factor 2 is w_2 . First, set the prices of both the inputs at 1.
 - (a) What can you say about the technical rate of substitution of the production function?
 - (b) What is the iso-cost function? What is the marginal product of the production function?
 - (c) Calculate the cheapest way to produce 16 units of output, showing clearly the demand for both the inputs and the associated cost.
 - (d) Show this analysis diagrammatically.

(You can practice by substituting different output objectives, input prices and production functions)

2. Geetha sells milkshakes in a competitive market on a corner of Buchanan street. Her production function is $f(x_1, x_2) = (x_1 x_2)^{1/3}$ where output is measured in gallons, x_1 is the containers of ice cream which is used and x_2 hours of labour hours spent to produce the milkshake.
 - (a) What can you say about the returns to scale of the production function?
 - (b) Where w_1 is the price of a container of ice cream and w_2 is the wage rate for milk shake maker write down the iso-cost function for cost C.
 - (c) Define and find the marginal product.
 - (d) Geetha estimates that she would be able to sell Y gallons of milkshakes. Calculate the production plan which would enable her to produce this amount in the cheapest way and the associated cost.
 - (e) Discuss what would happen to the production plan in (d) when there is an increase in (i) w_1 , (ii) w_2 and (iii) y .
3. For each production function, indicate whether has an increasing, decreasing or constant returns to scale. Then calculate the marginal product of each of the inputs.
(This is an exercise for you to practice the algebra – not to get stressed about!)

$f(x_1, x_2)$	Scale	MP_1	MP_2
$x_1 + 2x_2$			
$\sqrt{x_1 + 2x_2}$			
$.2x_1x_2^2$			
$x_1^{1/4}x_2^{3/4}$			
$x_1 + \sqrt{x_2}$			
$(x_1 + 1)^{-5}(x_2)^{-5}$			
$(x_1^{1/3} + x_2^{1/3})^3$			

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