

Tutorial 6 – Uncertainty

1. Portia has waited a long time for her ship to come in. She has concluded that it will arrive today with probability $\frac{1}{4}$. If it does come, she will receive \$16. If it does not come in today, it never will and she will have zero wealth. She has a von Neumann-Morgenstern utility function equal to the square root of her total income. Use an appropriate diagram and calculations to find Portia's expected wealth and expected utility. Explain the certainty equivalent in this case. Explain Portia's risk preference. What is the minimum price at which she would sell the rights to her ship?
2. Joe's wealth is £100 and he is an expected utility maximiser with a von-Neumann-Morgenstern utility function $U(W) = W^{1/2}$. He is afraid of oversleeping his economics exam. He estimates there is only a 1 in 10 chance that he will, but if he does, it will cost him \$100 in fees to the university for taking an exam late. Show the following analysis in a diagram with the utility on the y-axis and wealth on the x-axis.
 - Derive the expected loss and expected utility.
 - Joe's flatmate, Mary, never oversleeps. She offers to wake him one hour before the test, but he must pay her for this service. What is the certainty equivalent in this uncertain situation faced by Joe? What is the most he would be willing to pay for this wake-up service?
 - Explain why we can say that Joe is risk averse.
3. After graduating, Sallie Handshake's best job offer will either be with a large accounting firm for \$160,000 a year or a small accounting firm for \$40,000 a year. She can increase the probability of the former outcome by studying more, but such studying has its costs. If S represents her amount of studying (where $S = 0$ is no study and $S = 1$ is all-out effort), her probability of getting the job with a large firm is S . Therefore her utility depends on how hard she studies and her subsequent annual income Y , given by $U(S, Y) = Y^{1/2} - 400S^2$. Explain the features demonstrated by Sallie's utility function. Showing all your workings clearly, derive how much she has to study in order to maximise her expected utility.
4. Willy's only source of wealth is his chocolate factory. His preference function is given by $U(M) = \sqrt{M}$, when his wealth is M . His wealth, contingent on there being a flood and no flood are given by c_f and c_{nf} respectively. There is a probability p that there will be a flood.
 - (a) Write down Willy's expected utility function.

The value of the factory is £500,000 when there is no flood and zero if there is flood. The probability of there being a flood is $\frac{1}{13}$. Willy can buy insurance where if he buys £ x worth insurance he has to pay £ $3x/15$ regardless of there being flood or not.
 - (b) Indicate the endowment point (i.e. when there is no insurance) and draw his budget line (market opportunity line) on a diagram with c_{nf} on the x-axis and c_f on the y-axis. What is the slope of the line?
 - (c) What is the marginal rate of substitution?
 - (d) As a maximiser of expected utility, how much insurance should Willy buy?
5. This question is for general discussion in the classroom (i.e. I am not saying whether you are an EU maximiser or how you make your decision). You are thinking of entering a competition. A fair coin is tossed. If it comes up Heads the Bookie will give you £1000. However, if it comes up Tails, you will have to give the Bookie £1000. Explain whether you will participate in this lottery or not. Think about the factors which affect your decision.