ECON 2A: Level 2 Microeconomics

Cost Minimization

(Reading: Varian Chapter 21)

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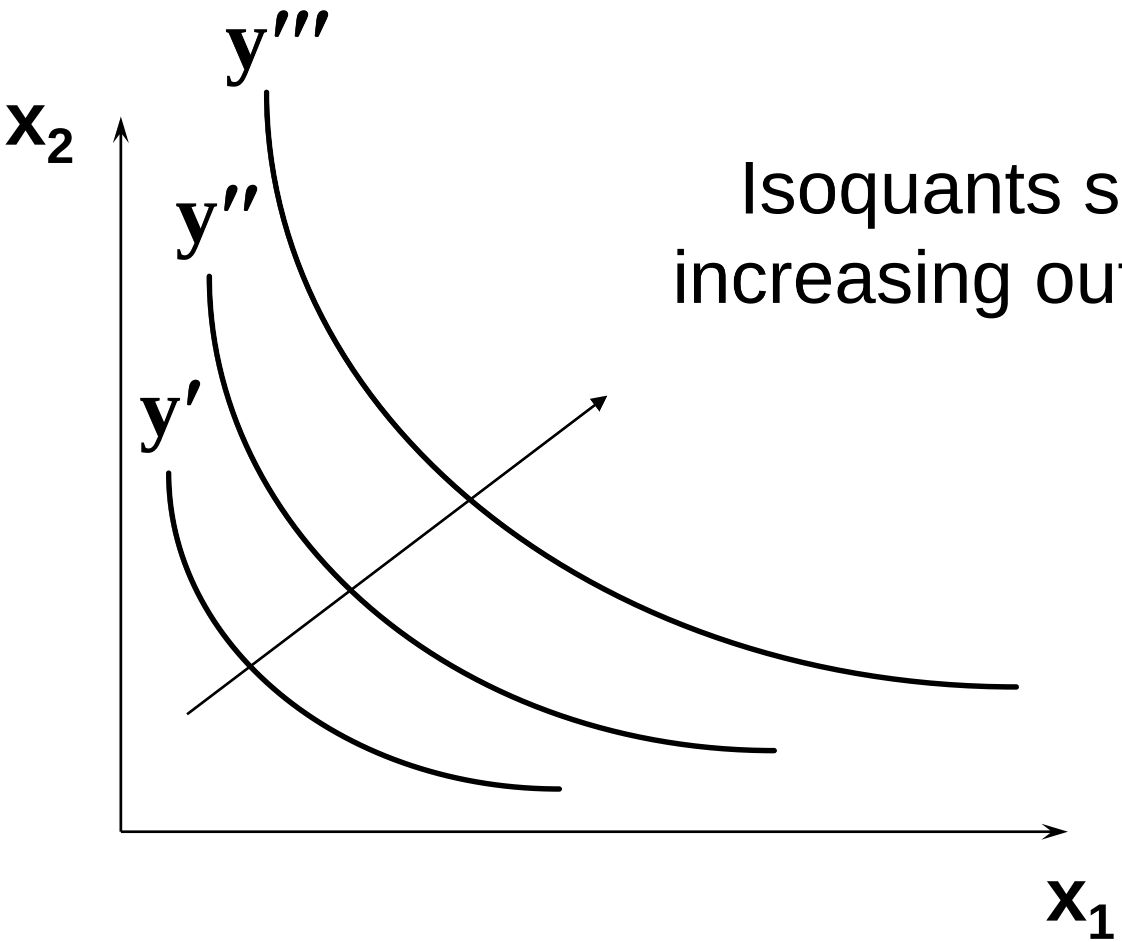
Consider a firm using two inputs to make one output.

The production function is $y = f(x_1, x_2)$.



Isoquants showing the increasing output levels …

A firm is a cost-minimizer if it produces any given output $y \geq 0$ at the smallest possible total cost.

Some notations

- y is the units of output produced.
- $c(y)$ is the total cost function producing y units of output
- $x = (x_1, x_2, \ldots, x_n)$ are the inputs

  $w = (w_1, w_2, \ldots, w_n)$ are the corresponding input prices.

Firm's Cost-Minimization problem:

- A firm uses two inputs to make one output
- Production function is $y = f(x_1, x_2)$.
- Given the input prices w_1 and w_2,

  the cost of an input bundle (x_1, x_2) is $w_1x_1 + w_2x_2$.
- Take the output level $y \geq 0$ as **given**.

For given w_1, w_2 and y, **the cost-minimization problem:**

$$\min_{x_1, x_2 \geq 0} w_1 x_1 + w_2 x_2 \quad \text{subject to} \quad f(x_1, x_2) = y.$$

- Solve the minimization problem and find the levels $x_1^*(w_1,w_2,y)$ and $x_1^*(w_1,w_2,y)$ as the least-costly input bundle, which are the firm's conditional demands for inputs 1 and 2.

- The (smallest possible) total cost for producing y output units is therefore

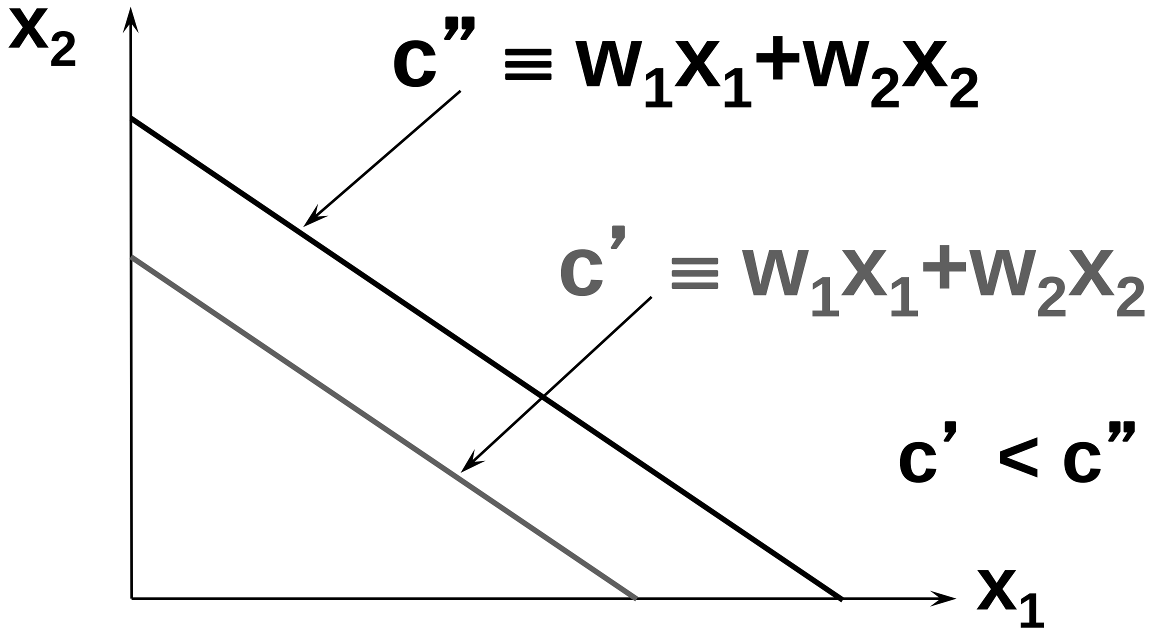
Total Cost function

$$c(w_1, w_2, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y).$$

Iso-cost Lines

A curve that contains all of the input bundles that cost the same amount is an iso-cost curve.

$$w_1 x_1 + w_2 x_2 = c \quad \Rightarrow x_2 = -\frac{w_1}{w_2} x_1 + \frac{c}{w_2}.$$



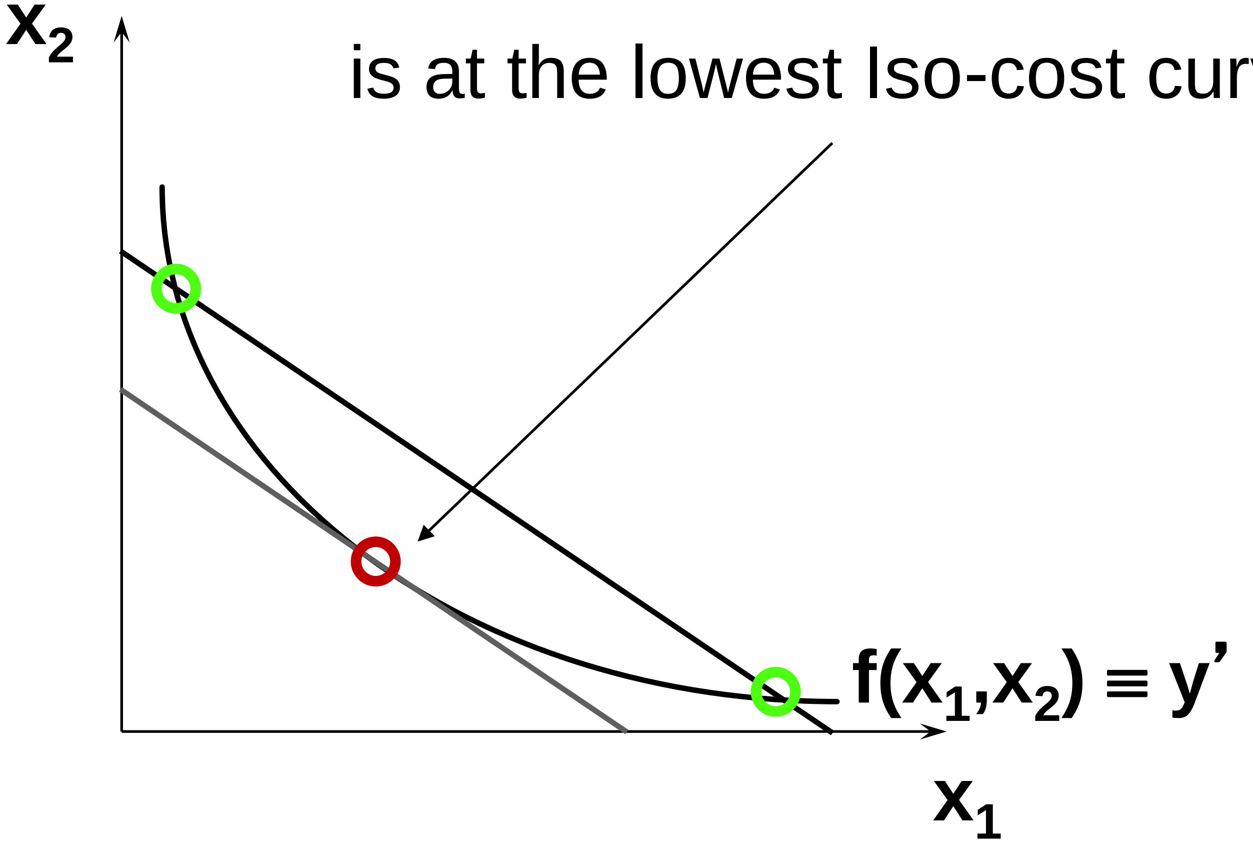
Slopes of Isocost

$= -w_1/w_2$.

All input bundles yielding y' units of output. $f(x_1,x_2) \equiv y'$

The cheapest bundle which produces y'

is at the lowest Iso-cost curve.

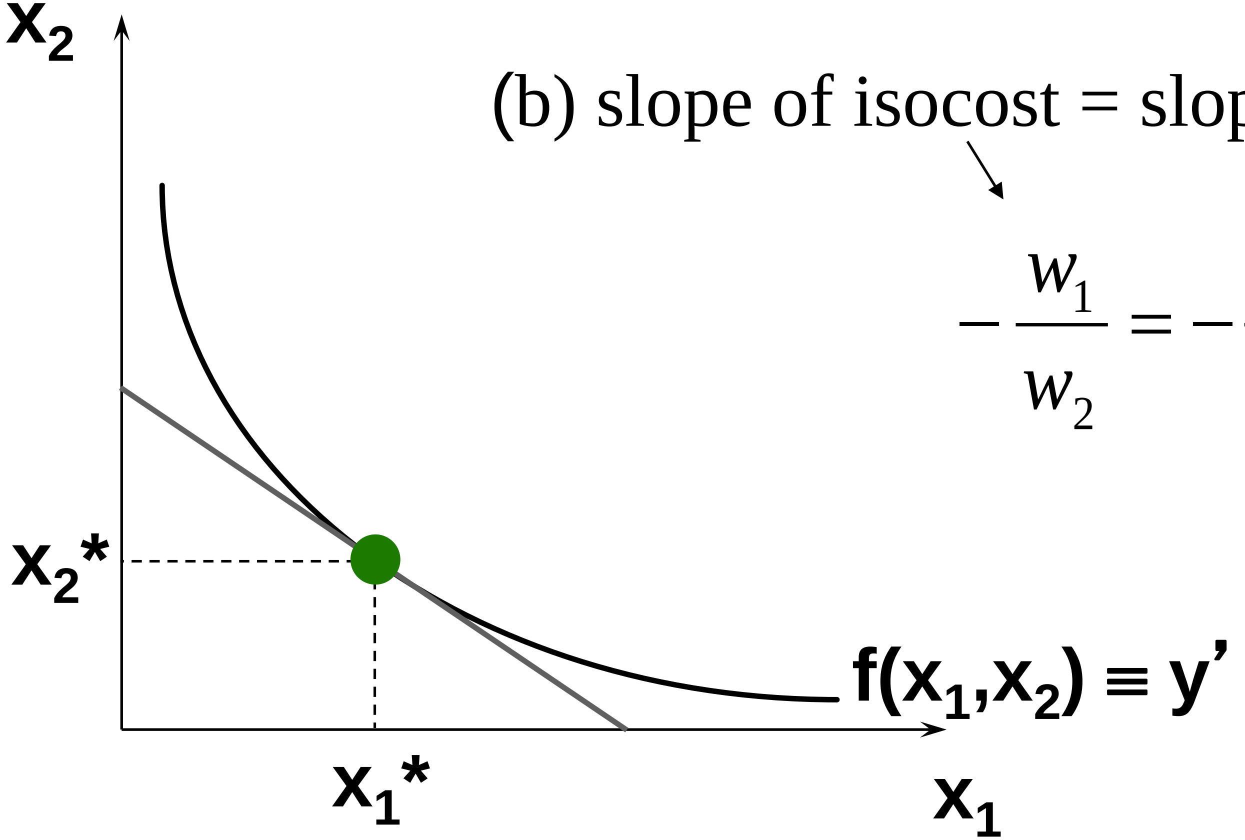


cost-minimising input bundle:

$$(a) \quad f(x_1^*, x_2^*) = y'$$

(b) slope of isocost = slope of isoquant

$$-\frac{w_1}{w_2} = -\frac{MP_1}{MP_2}.$$



A Cobb-Douglas Example

A firm's production function is given by

$$y = f(x_1, x_2) = x_1^{1/3} x_2^{2/3}.$$

Input prices are w_1 and w_2.

At the optimum point,

slope of isocost = slope of isoquant ➔ $-\dfrac{w_1}{w_2} = -\dfrac{MP_1}{MP_2}$

$$-\frac{MP_1}{MP_2} = -\frac{\partial y / \partial x_1}{\partial y / \partial x_2}$$

$$= -\frac{(1/3)(x_1^*)^{-2/3}(x_2^*)^{2/3}}{(2/3)(x_1^*)^{1/3}(x_2^*)^{-1/3}} = -\frac{x_2^*}{2x_1^*}.$$

$$-\frac{w_1}{w_2} = -\frac{x_2^*}{2x_1^*}. \qquad \Rightarrow x_2^* = \frac{2w_1}{w_2} x_1^*.$$

Substitute this into $y = (x_1^*)^{1/3}(x_2^*)^{2/3}$

$$\mathbf{y = (x_1^*)^{1/3}\left(\frac{2w_1}{w_2}x_1^*\right)^{2/3} = \left(\frac{2w_1}{w_2}\right)^{2/3} x_1^*.}$$

So $$\boxed{\mathbf{x_1^* = \left(\frac{w_2}{2w_1}\right)^{2/3} y}}$$

Substitute this into $x_2^* = \dfrac{2w_1}{w_2} x_1^*,$

$$\boxed{x_2^* = \left(\frac{2w_1}{w_2}\right)^{1/3} y}$$

Some comparative statics analysis:

The cheapest input bundle yielding y output units:

$$\mathbf{x_1^* = \left(\frac{w_2}{2w_1}\right)^{2/3} y}$$

$$x_2^* = \left(\frac{2w_1}{w_2}\right)^{1/3} y$$

Notice and discuss intuition that the inputs,

- increase with y,

- decrease with own price

- increase with price of other input

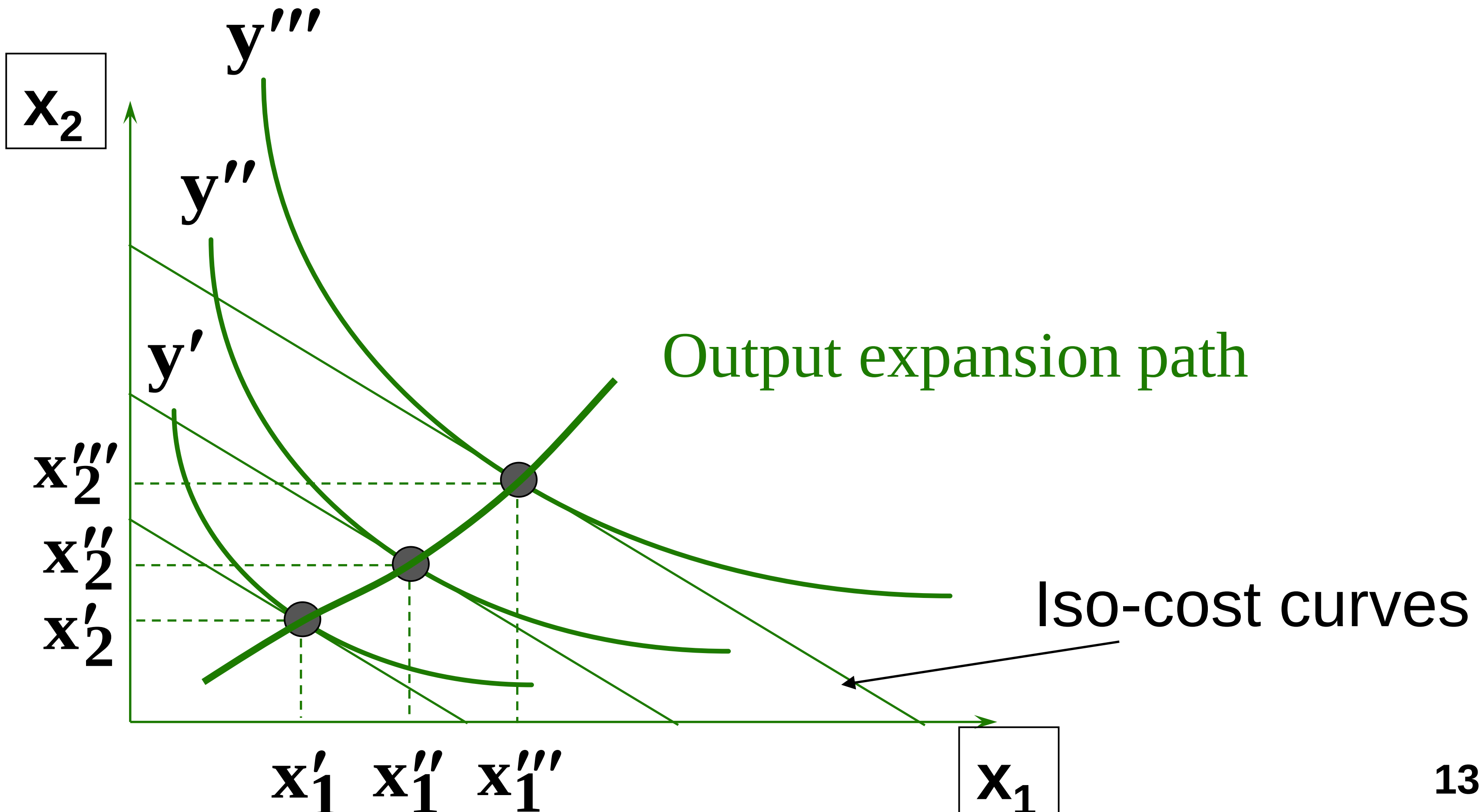
The firm's total cost function is

$$c(w_1, w_2, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y)$$

$$= w_1 \left(\frac{w_2}{2w_1} \right)^{2/3} y + w_2 \left(\frac{2w_1}{w_2} \right)^{1/3} y$$

$$= \left(\frac{1}{2} \right)^{2/3} w_1^{1/3} w_2^{2/3} y + 2^{1/3} w_1^{1/3} w_2^{2/3} y$$

$$= 3 \left(\frac{w_1 w_2^2}{4} \right)^{1/3} y.$$


x2
y'''
y''
y'
Output expansion path
x2'''
x2''
x2'
Iso-cost curves
x1'
x1''
x1'''
x1

Average Total Production Costs

For positive output levels y, a firm's average total cost of producing y units is

$$AC(w_1, w_2, y) = \frac{c(w_1, w_2, y)}{y}.$$

If a firm's technology exhibits **constant returns-to-scale** then doubling its output level from y' to 2y' requires doubling all input levels.

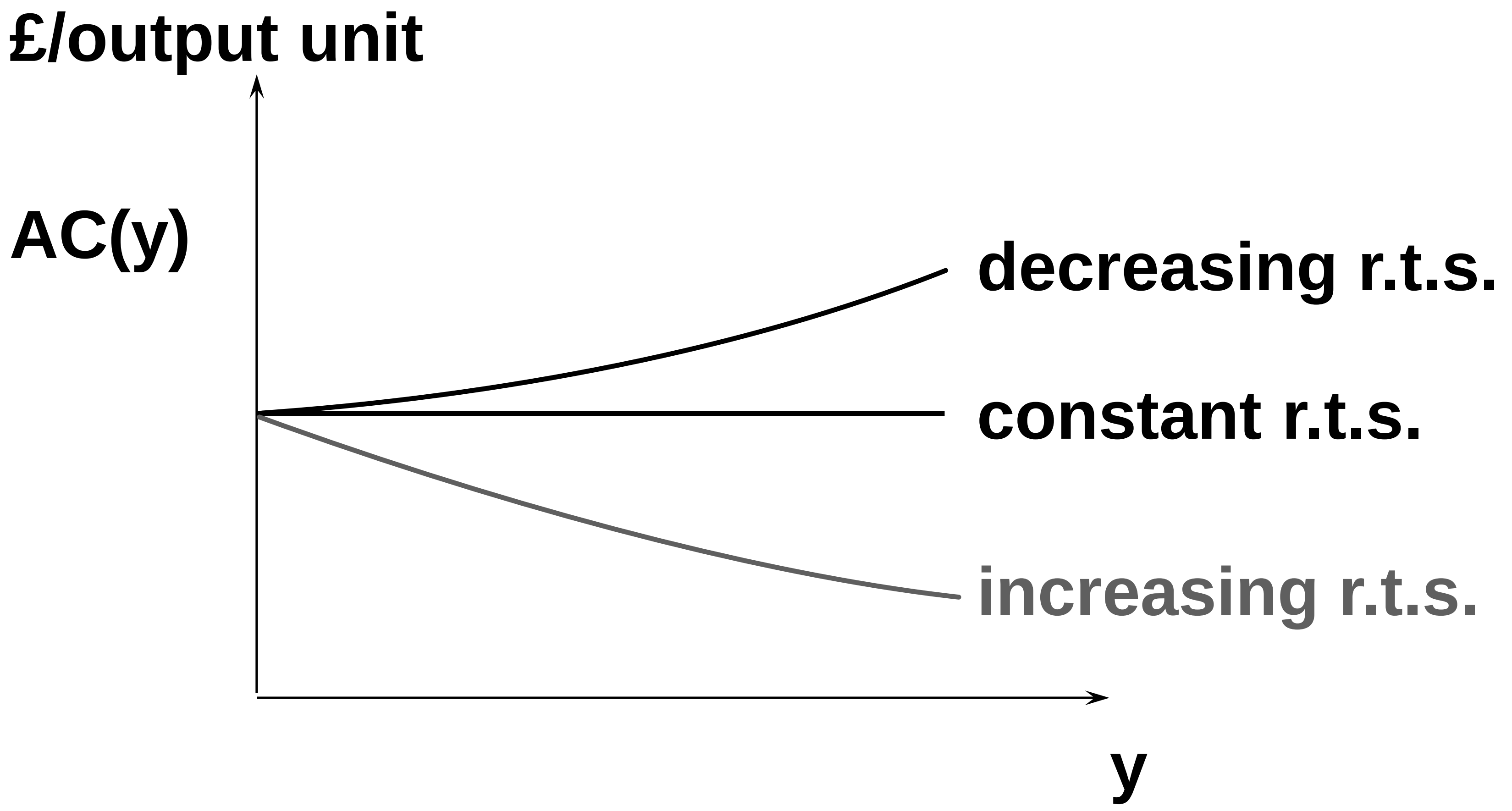
- Total production cost doubles, output also doubles.
- Average production cost does not change.

If a firm's technology exhibits **decreasing returns-to-scale,** then doubling its output level from y' to 2y' requires more than doubling all input levels.

- Total production cost more than doubles, output doubles.
- Average production cost increases.

If a firm's technology exhibits **increasing returns-to-scale** then doubling its output level from y' to 2y' requires less than doubling all input levels.

- Total production cost less than doubles, output doubles.
- Average production cost decreases.


£/output unit
AC(y)
decreasing r.t.s.
constant r.t.s.
increasing r.t.s.
y