ECON 2A: Microeconomics

Uncertainty (2&3)

Reading: Varian Chapter 12

From the last class…

Von Neumann-Morgenstern utility function

This is an expected utility function under uncertainty – when playing a lottery.

Geetha is thinking of betting on Roger Federer winning the ATP World Tour Finals.

She thinks the probability that he will not win is π_1

and will win is π_2.

If she does not bet, she is **certain** to retain what she would have spent on betting.

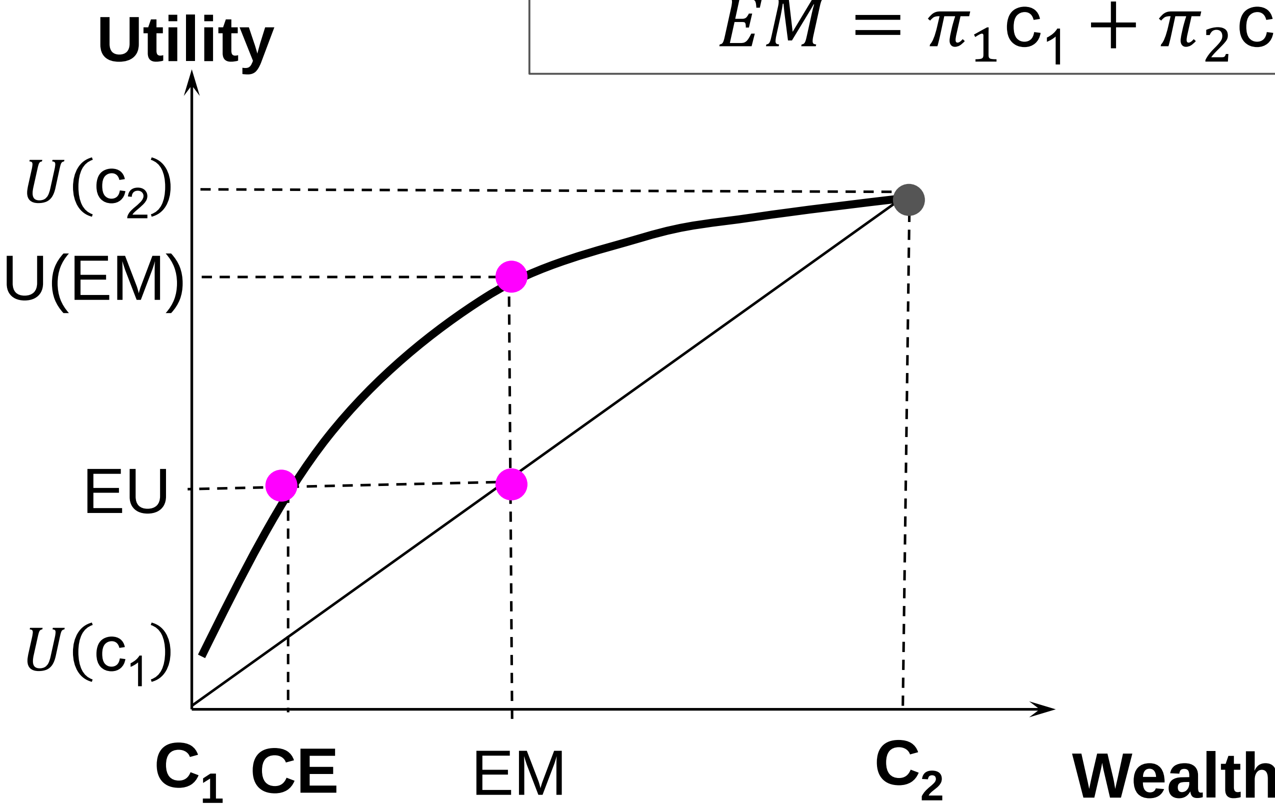
She gains c_2 if she wins the bet, and c_1 if she loses.

Expected utility function

(or the von-Neumann-Morgenstern utility function):

$$EU = \pi_1 U(c_1) + \pi_2 U(c_2)$$

$$EM = \pi_1 c_1 + \pi_2 c_2$$

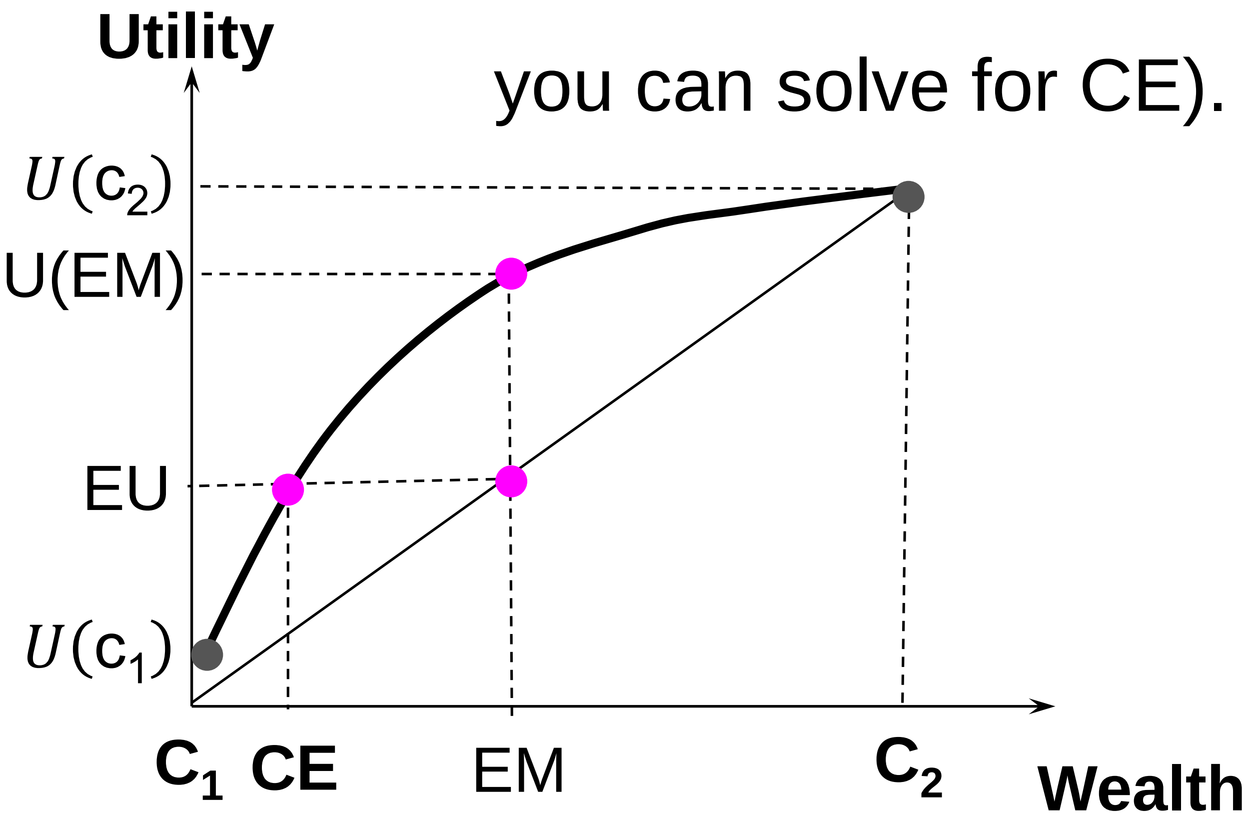


Certainty Equivalent (CE) is the value which will give the same utility as the expected utility.

U(CE)=EU

$$U(CE) = \pi_1 U(c_1) + \pi_2 U(c_2)$$

(If you know the functional form of U(.), you can solve for CE).



Example:

Johnny has £2500 in his bank account. He is invited for an interview in London for a one-off consultancy job. He has to spend £475 to attend this. He estimates that the probability of getting the job is 10%. If he gets the job, he will receive £3600. If he does not get the job, he just returns home with nothing.

If he decides not to attend the interview, he retains the £475 he would have spent. Johnny's preference is given by $\mathrm{u(X)} = \sqrt{X}$.

What will he decide ? Explain.

Calculate and show diagrammatically, the EU, CE, and his decision.

What is Johnny's risk preference – i.e. is he risk averse, risk loving or risk neutral?

You can do the same question - if the probability of winning is 40%.

- $\mathrm{u(X)} = X^2$.

Preferences Under Uncertainty

MRS of an indifference curve under uncertainty

(Marginal Rate of Substitution)

- Let consumption be: c_1 with probability π_1

  and c_2 with probability π_2

  (where $\pi_1 + \pi_2 = 1$).

- **$EU = \pi_1 U(c_1) + \pi_2 U(c_2)$.**

$$EU = \pi_1 U(c_1) + \pi_2 U(c_2).$$

First, we calculate the marginal utilities:

$$MU(c_1) = \frac{dU(c_1)}{dc_1} \Rightarrow dU(c_1) = MU(c_1)dc_1$$

$$MU(c_2) = \frac{dU(c_2)}{dc_2} \Rightarrow dU(c_2) = MU(c_2)dc_2$$

Next: We know that for constant EU (i.e. no change in EU),

dEU = 0

$$dEU = \pi_1 dU(c_1) + \pi_2 dU(c_2)$$

$$dEU = \pi_1 MU(c_1)dc_1 + \pi_2 MU(c_2)dc_2$$

$$\pi_1 MU(c_1)dc_1 + \pi_2 MU(c_2)dc_2 = 0$$

$$\Rightarrow \pi_1 MU(c_1)dc_1 = -\pi_2 MU(c_2)dc_2$$

$$\boxed{\Rightarrow \frac{dc_2}{dc_1} = -\frac{\pi_1 MU(c_1)}{\pi_2 MU(c_2)},}$$

which is the MRS

(marginal rate of substitution)

Concepts affecting analysis of 'Uncertain' scenarios:

- States of Nature which are possible:

  eg - car accident, a;

  - no car accident, na

- Probability of Accident - occurring: π_a

  - not occurring: π_{na}

$$\pi_a + \pi_{na} = 1$$

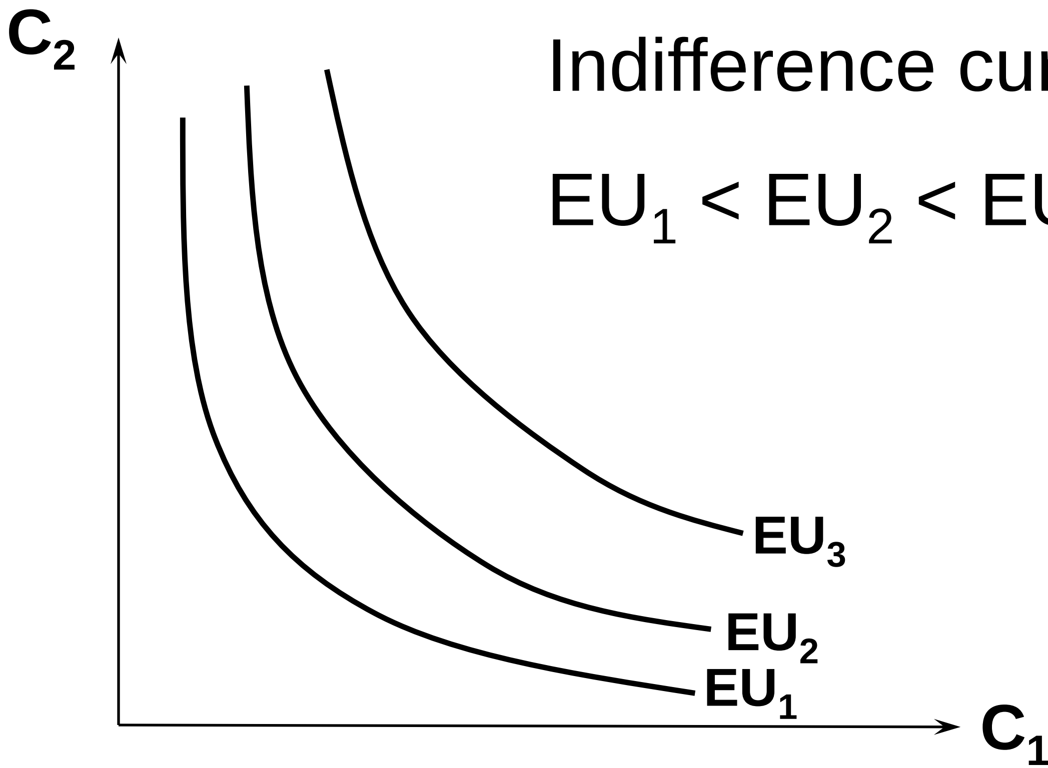
- Outcome: Accident causes a **loss of $£L$**

- State-contingent Contract is a contract implemented only when a particular state of Nature occurs.

  E.g. An insurance contract where the insurer pays only if there is an accident.

- A state-contingent consumption the value only when a particular state of Nature occurs.

State-contingent consumption plans that give equal expected utility are equally preferred.



Indifference curves

$EU_1 < EU_2 < EU_3$

(in our example, c_1 and c_2 will be c_a and c_{na}.)

Consider the following uncertain scenario:

- Each £1 of accident insurance costs γ

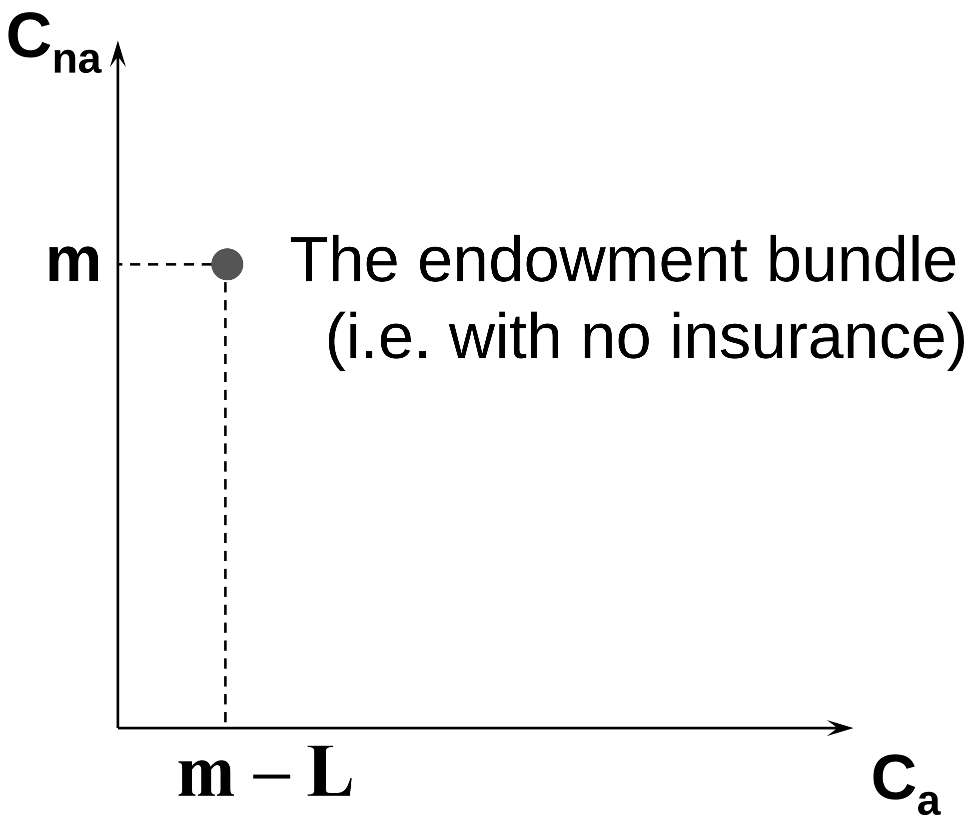
  (i.e. insurance premium is γ per £1 to be received)

- Consumer has wealth of £m.
- Consumption value in the state of accident is C_a.
- Consumption value in the state of no-accident is C_{na}.
- Probability of state of accident is π_a

  and no accident is $\pi_{na} = 1 - \pi_a$

Outcome without insurance (endowment bundle)

- $C_{na} = m$.
- $C_a = m - L$



Outcome with Insurance

If £K of accident insurance is bought – i.e. premium γK :

$$C_{na} = m - \gamma K \qquad (1)$$

$$C_a = m - \gamma K - L + K \qquad (2)$$

Rearranging (2) ➔ $C_a - m + L = K - \gamma K$

➔ $C_a - m + L = K(1-\gamma)$

So $K = \dfrac{(C_a - m + L)}{(1-\gamma)}$

Substituting K into (1), $C_{na} = m - \gamma \dfrac{(C_a - m + L)}{(1-\gamma)}$

State-Contingent Budget Constraints

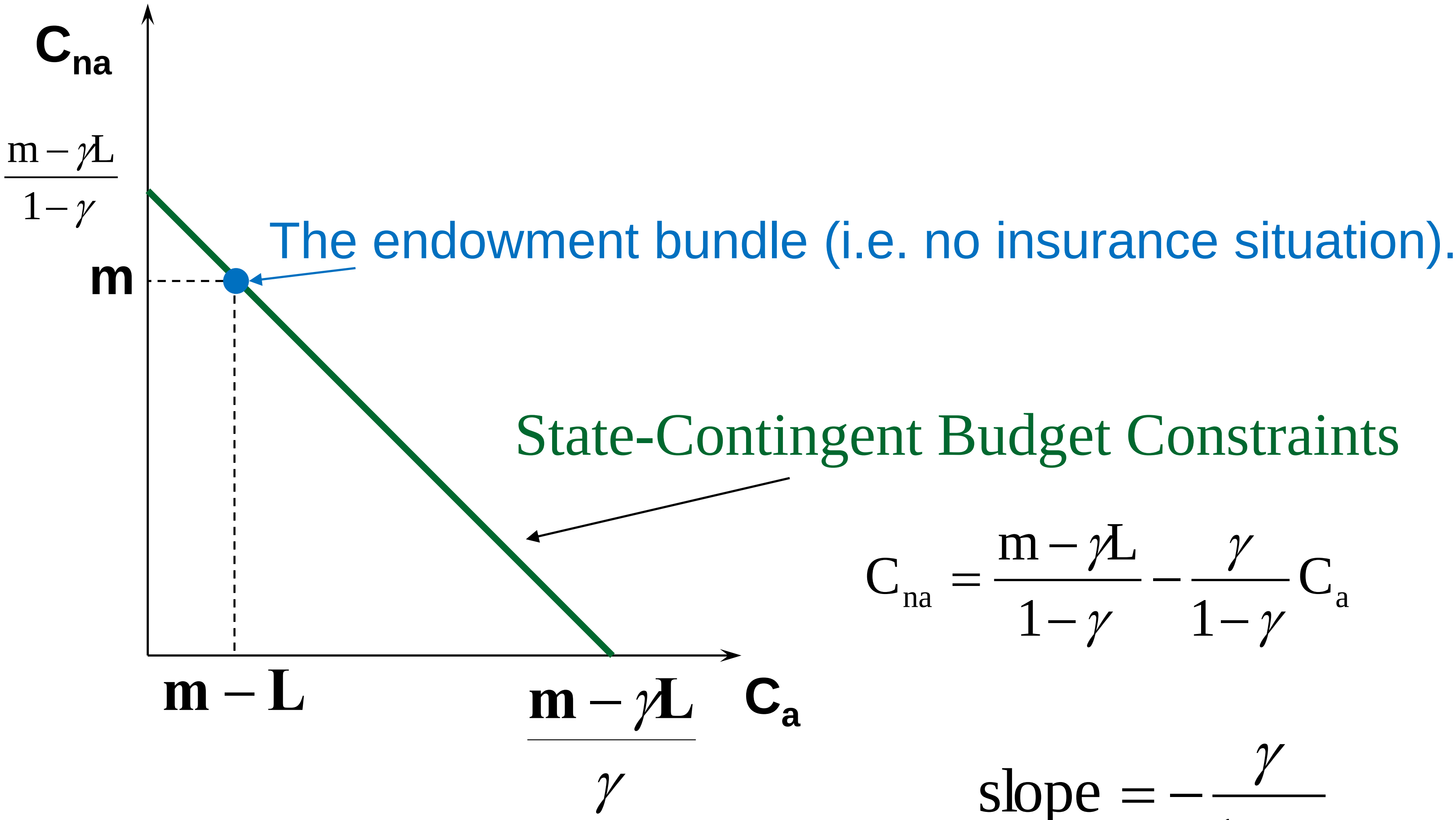
From the previous slide,

$$C_{na} = m - \gamma \frac{(C_a - m + L)}{(1-\gamma)}$$

Rearranging, $C_{na} = \frac{m(1-\gamma)}{(1-\gamma)} - \frac{(\gamma C_a - \gamma m + \gamma L)}{(1-\gamma)}$

$$C_{na} = \frac{m - m\gamma + m\gamma - \gamma L}{(1-\gamma)} - \frac{\gamma C_a}{(1-\gamma)}$$

$$\boxed{\mathbf{C_{na}} = \frac{\mathbf{m} - \gamma\mathbf{L}}{\mathbf{1} - \gamma} - \frac{\gamma}{\mathbf{1} - \gamma}\mathbf{C_a}}$$



$$C_{na} = \frac{m-\gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} C_a$$

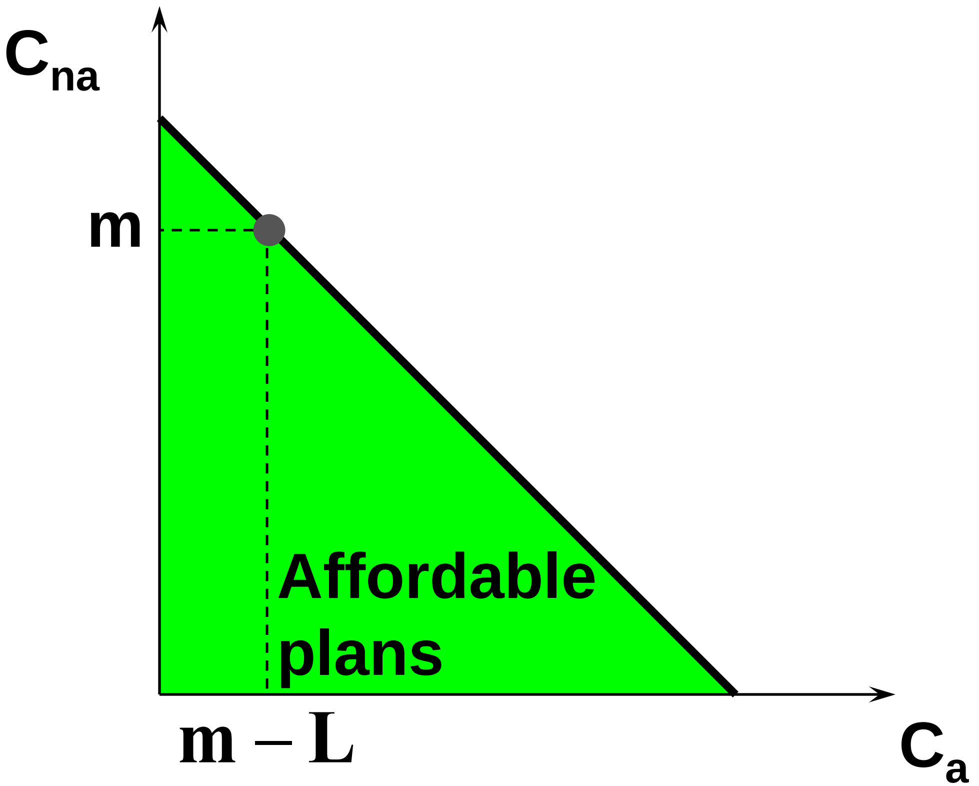
$$\text{slope} = -\frac{\gamma}{1-\gamma}$$

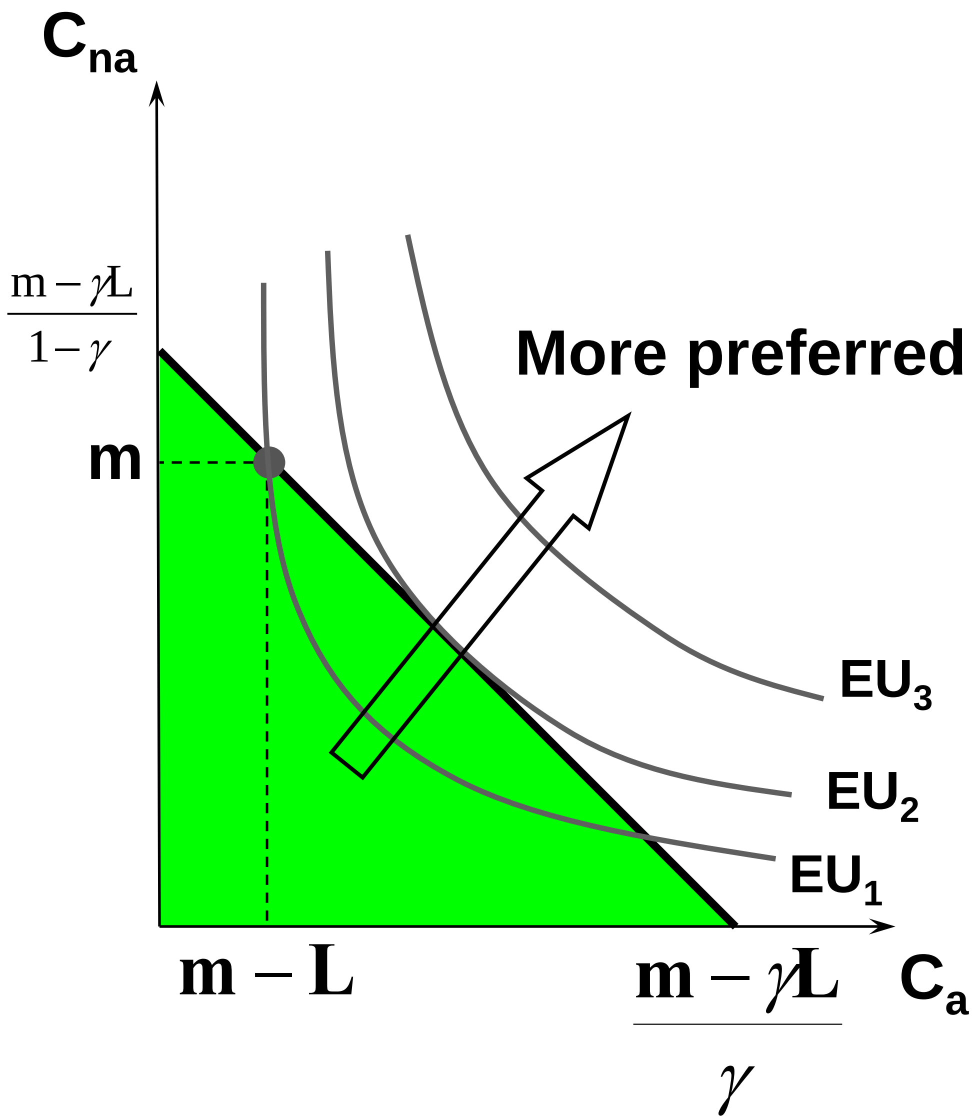
Where is the most preferred state-contingent consumption plan?

Choice Under Uncertainty

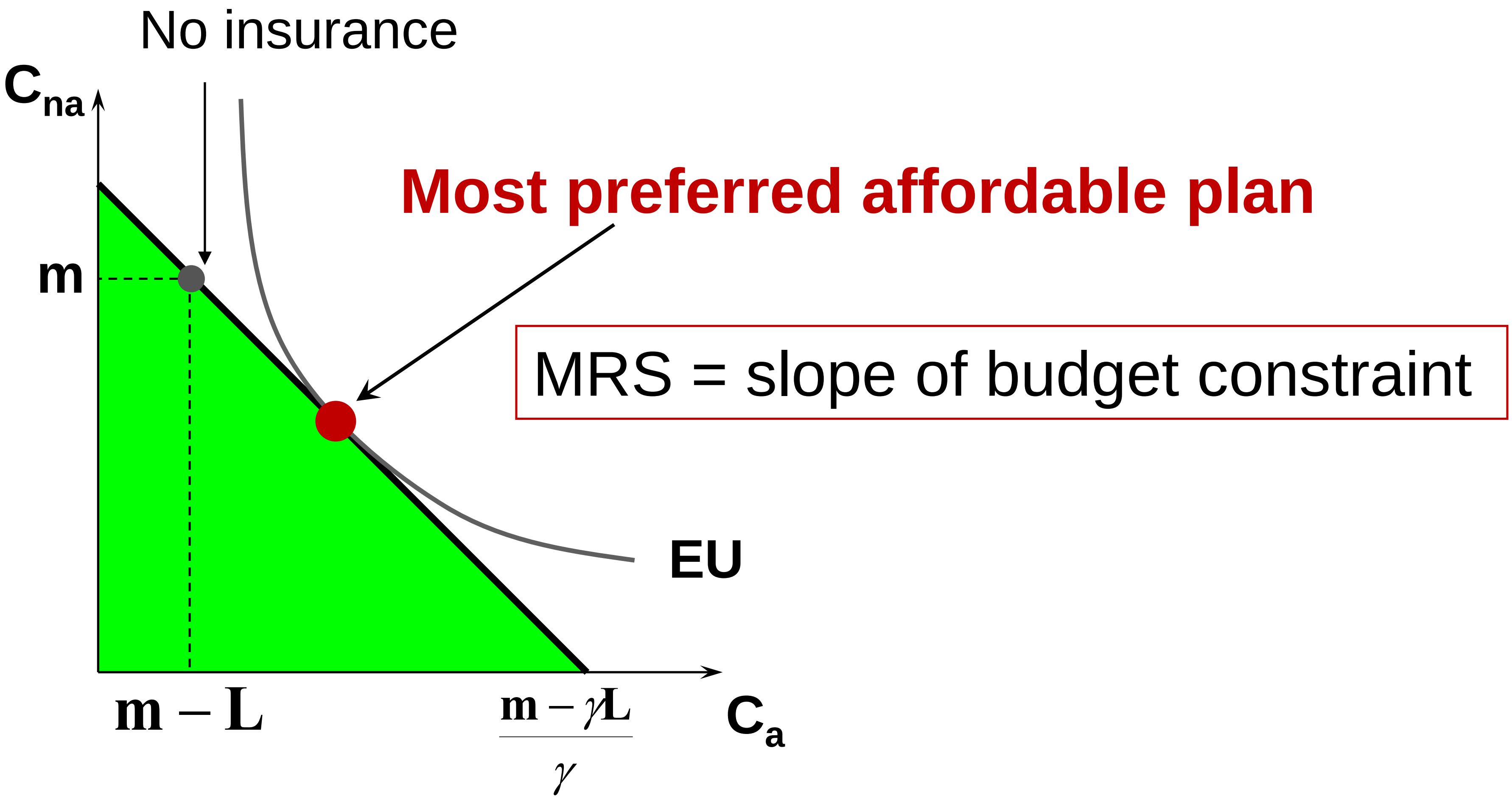
Choose the most preferred plan which is affordable

i.e. maximize **expected utility** subject to the **state-contingent budget constraint**.





C_na
(m − γL) / (1 − γ)
m
More preferred
EU_3
EU_2
EU_1
m − L
(m − γL) / γ
C_a



(Recall: MRS is the slope of the utility function.)

In order to get the most preferred affordable plan (optimum choice), an insurance can be obtained.

Example:

Geetha has wealth of £5000 and drives a car She estimates there is a 0.25 probability of an accident.

If there is an accident, she will lose £3000. Each £1 of accident insurance costs 20 pence (i.e. £0.2)

Geetha's utility function is $\mathrm{U}(\mathrm{X}) = logX$. Analyse her decision making process.