EC109: Microeconomics I

University of Warwick
Department of Economics

Terms 1 and 2

The 2 year road map

- The plan for your 2 years in microeconomics
  - The core topics; The core skills; The tools of the trade

EC109

- Consumer Theory
- Producer Theory
- Market structure and firm behaviour
- Intro to Game Theory
- Partial equilibrium

EC202

- Choice under Uncertainty
- Game Theory
- General equilibrium
- Market failure

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CONSUMER THEORY

The Topics

- Budget Constraints and the feasible set
- Preferences
- Indifference curves and utility functions
- Revealed Preferences
- Optimisation
- Comparative statics
- Changes in welfare
- Applications

An example

- If you and I go shopping in our respective local towns, why is it unlikely we will each come out of the shop with the same amount of each good in our baskets?
- If I buy more milk than you, what are the possible explanations?
- Differences in behaviour can emerge from
  - Different tastes
  - Different circumstances
- We optimise subject to our constraints

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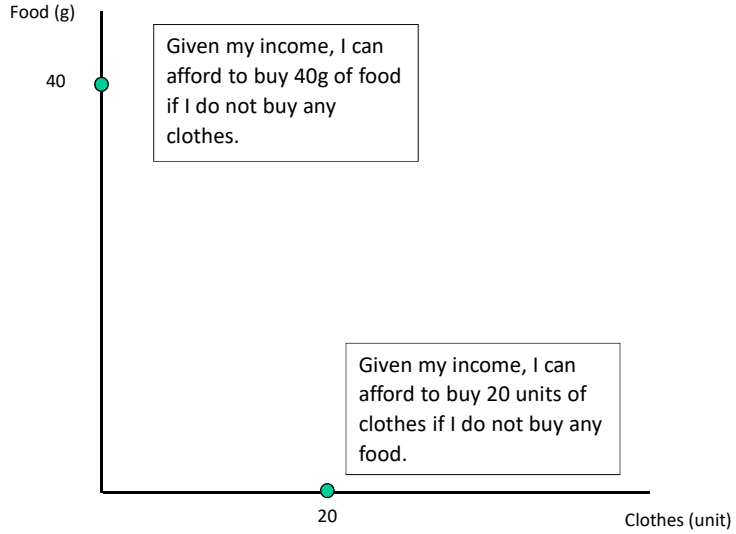
BUDGET CONSTRAINTS

Budget constraint I

- Income and prices affect the quantity consumers demand
  - Income can be determined exogenously as an amount, M
  - Or determined endogenously from resources
- Assume my weekly income is £200 and I spend money on food at £5/g and clothes at £10/unit.
  - M= $F \times P_f$ if I decide to only consume food.
    - $200 = F \times 5 \rightarrow F = 40 = \frac{M}{P_f}$
  - M= $C \times P_c$ if I decide to only buy clothes.
    - $200 = C \times 10 \rightarrow C = 20 = \frac{M}{P_c}$

Budget constraint II



Budget Constraint III

- If I want more balanced consumption, it must be true that I do not spend more than my income:

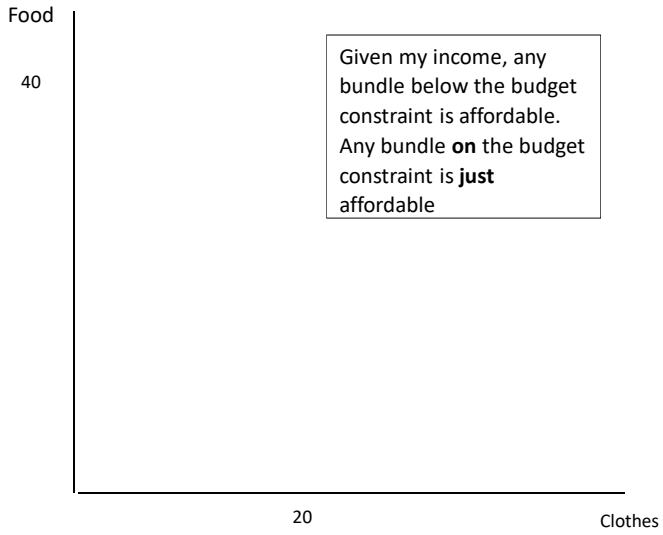
$$M \geq F \times P_f + C \times P_c$$

- With food on the vertical axis, we can rewrite the budget constraint as:

$$F = \frac{M}{P_f} - \frac{P_c}{P_f} C$$

- We can now determine the vertical intercept and slope of the budget constraint

The feasible set



Given my income, any bundle below the budget constraint is affordable. Any bundle **on** the budget constraint is **just** affordable

Plugging in the values for M, P_f and P_c allows us to construct the budget constraint and determine the feasible set:

$$200 = 5F + 10C$$

$$F = \frac{200}{5} - \frac{10}{5}C$$

Budget constraint IV

- The slope measures the rate the market 'substitutes' good 1 for good 2: it's the opportunity cost of consuming good 1
- If we consume more good 1, Δx_1, by how much must good 2 change to continue to satisfy the budget constraint?

$$p_1 x_1 + p_2 x_2 = m \qquad (1)$$

$$p_1(x_1 + \Delta x_1) + p_2(x_2 + \Delta x_2) = m \qquad (2)$$

- Subtract (1) from (2) to find: $p_1 \Delta x_1 + p_2 \Delta x_2 = 0$
- Thus: $\frac{\Delta x_2}{\Delta x_1} = -\frac{p_1}{p_2}$

Changing prices and income

Returning in each case to $M = £200;\ P_f = £5;\ P_c = £10$

- Income falls to £100
- Price of clothes falls from £10 to £8
- Both prices double
- Prices of food and clothes rise and income falls

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PREFERENCES

Preferences

- We assume consumers choose what they want the most
- Specifying preferences tells us something about a consumer's choice.
- Consider the bundle (x_1, x_2) and compare it with (y_1, y_2) to determine the preference ordering:
  - Strict preference $\succ$
  - Weak preference $\succcurlyeq$
  - Indifference $\backsim$
- We only care about ordinal relations

Properties of Preferences

- Completeness
  - The consumer can always compare/rank bundles. Either $X \succ Y, or\ Y \succ X, or\ X \backsim Y$
- Transitivity
  - If $X \succcurlyeq Y$ and $Y \succcurlyeq Z$ then $X \succcurlyeq Z$
- Continuous
  - If X is preferred to Y, and there is a third bundle Z which lies within a small radius of Y, then X will be preferred to Z.
  - Tiny changes in bundles will not change preference ordering

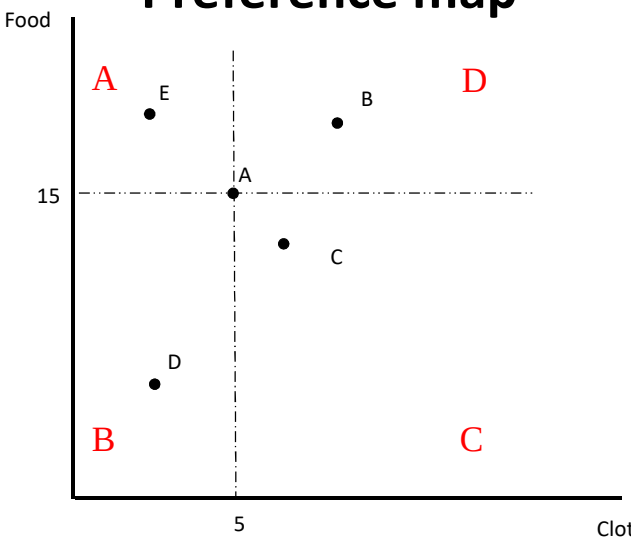
Well-behaved preferences

- Monotonicity (non-satiation)
  - We are talking about *goods* and not *bads* >> More is better!
  - Consider two bundles X and Y. If Y has at least as much of both goods, and more of one, then $(y_1, y_2) \succ (x_1, x_2)$
- Convexity
  - Averages are better than extremes (or at least not worse)
  - An average of two bundles on the same indifference curve will be (at least weakly) preferred, for any $0 < t < 1$

$$z = (tx_1 + (1-t)y_1, tx_2 + (1-t)y_2) \succcurlyeq (x_1, x_2)$$

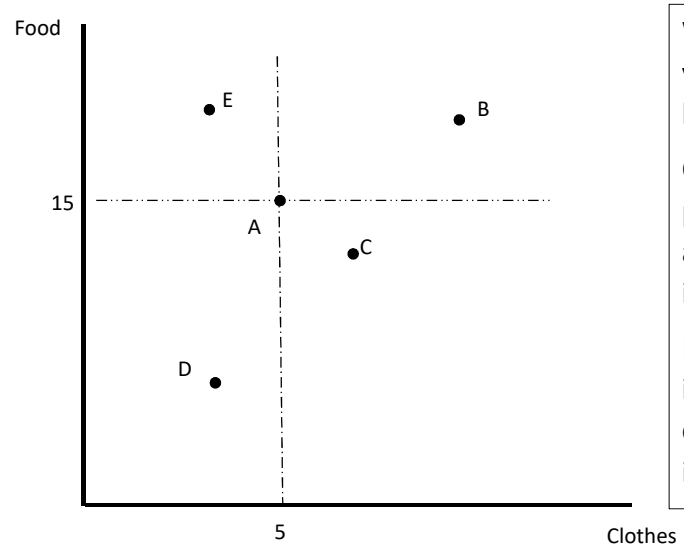
Preference map

More is better tells us …



Can we use more is better property to compare bundles A, C, E?

Indifference curves I

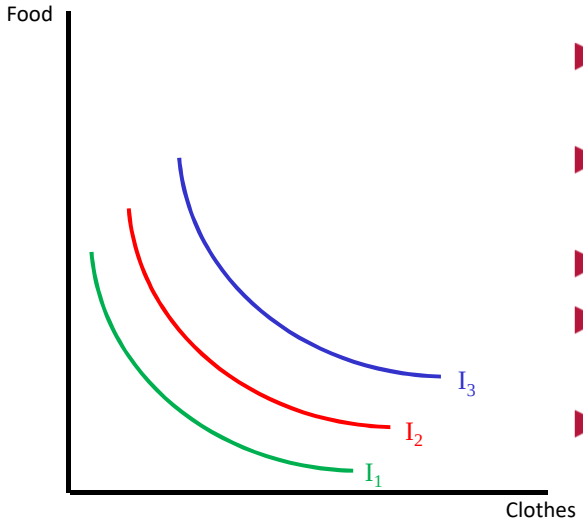


Which bundles do you like equally to bundle A?

Connecting these points (bundles A, C and E) creates an indifference curve.

Plotting other indifference curves creates an indifference map.

Indifference Curves II



- Bundles on I_3 are preferred to bundles on I_2 etc.
- Indifference curves are continuous
- Indifference curves cannot cross
- Most people's indifference curves are convex to the origin
- Indifference curves are downward sloping

Indifference curves III

- Perfect Substitutes – goods with a constant rate of substitution
- Perfect Complements – goods that are always consumed together
- 'Bads' – a good that you dislike
- Neutral goods – a good that you don't care about
- Satiation – an overall best bundle: too much AND too little is worse

Utility I

- Utility Functions describe preferences, assigning higher numbers to more-preferred bundles
  - All combinations of two goods that give an individual the same level of utility lie on the same indifference curve
- The further from the origin, the higher the utility
- Bundle $(x_1, x_2) \succ (y_1, y_2)$ iff $u(x_1, x_2) \succ u(y_1, y_2)$
- We are interested in ordinal and not cardinal utility
  - We care about which bundle is preferred and not by how much

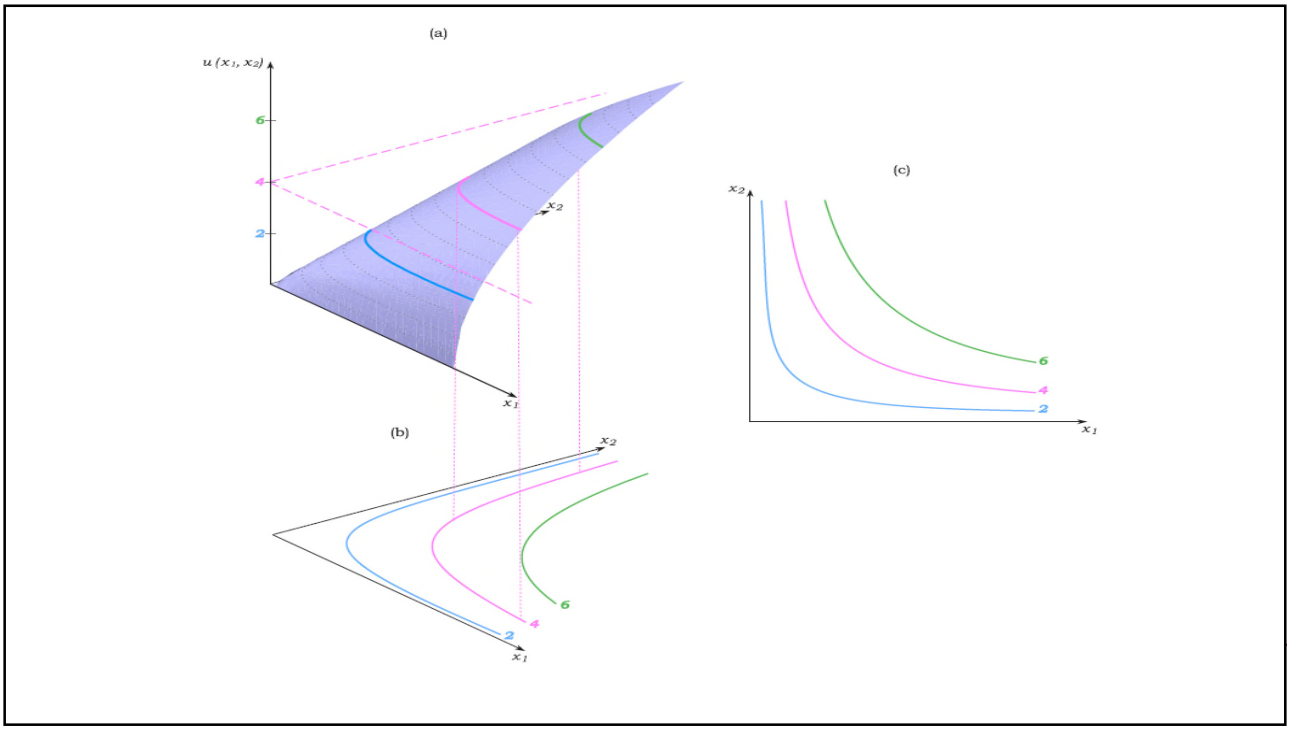
Monotonic transformations

- Applying monotonic transformations to a utility function creates a new function but with the same preferences
  - Transforms a set of numbers into another set; preserving the order
  - We can't work back from optimal demands for exact utility function
- Consider a utility function $U(x_1, x_2, \dots, x_n)$. Some examples of monotonic transformations:
  - $\log(U(x_1, x_2, \dots, x_n))$
  - $-\exp\big(U(x_1, x_2, \dots, x_n)\big)$
  - $U(x_1, x_2, \dots, x_n) + k$
  - $-\gamma(U(x_1, x_2, \dots, x_n))$
  - $U(x_1, x_2, \dots, x_n)^{\mathrm{n}}$
  - $-\sqrt{U(x_1, x_2, \dots, x_n)}$

Utility II

What will the utility functions look like for …

- Perfect Substitutes:
- Perfect Complements:
- Cobb-Douglas
  - Between the two extremes: Imperfect Substitutes
  - The simplest example of well-behaved preferences
  - $U(x_1, x_2) = x_1 x_2$ (or $U(x_1, x_2) = {x_1}^2 {x_2}^2$)
  - $U(x_1, x_2) = x_1^{\frac{3}{4}} x_2^{\frac{1}{4}}$
  - More generally: $U(x_1, x_2) = x_1^{\alpha} {x_2}^{1-\alpha}$



Behavioural insights I

Which factors affect utility?

- Psychological attitudes
- Peer group pressures
- Personal experiences
- The general cultural environment

Ceteris paribus

- Only consider choices among quantifiable options
- Hold constant other things that affect behaviour

Behavioural insights II

Do the axioms always hold? Are consumers truly rational?

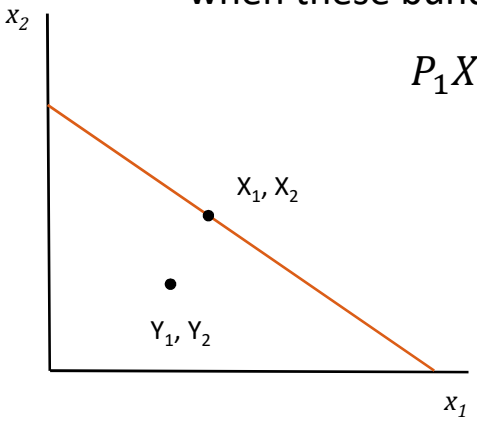
- Too many choices/much information; how choices are framed: Prospect Theory
- Loss Aversion: the disutility of giving up an object is greater than the utility associated with acquiring it (cognitive bias)
- What's the default option? (e.g. buying something online)

- Bounded rationality
  - Behaviour is influenced by our environment and the information we have
  - Poor feedback restricts information

Can behaviour be influenced to become more rational?

Revealed Preferences

If an optimising consumer chooses (X_1, X_2) over (Y_1, Y_2), when these bundles are different, it must be that:



$$P_1X_1 + P_2X_2 = M \quad \text{and} \quad P_1Y_1 + P_2Y_2 \leq M$$

$$P_1X_1 + P_2X_2 \geq P_1Y_1 + P_2Y_2$$

If this inequality is satisfied, then (X_1, X_2) is directly revealed preferred:

$$(X_1, X_2) \succ (Y_1, Y_2)$$

WARP and SARP

- If $(X_1, X_2) \succ (Y_1, Y_2)$, then it can't be that $(Y_1, Y_2) \succ (X_1, X_2)$
  - WARP refers to directly revealed preferences
  - SARP refers to directly and indirectly revealed preferences
- Say that at prices (P_1, P_2), bundle (X_1, X_2) is bought when (Y_1, Y_2) is affordable
- This means that if (Y_1, Y_2) is purchased at prices (Q_1, Q_2), then (X_1, X_2) must be unaffordable

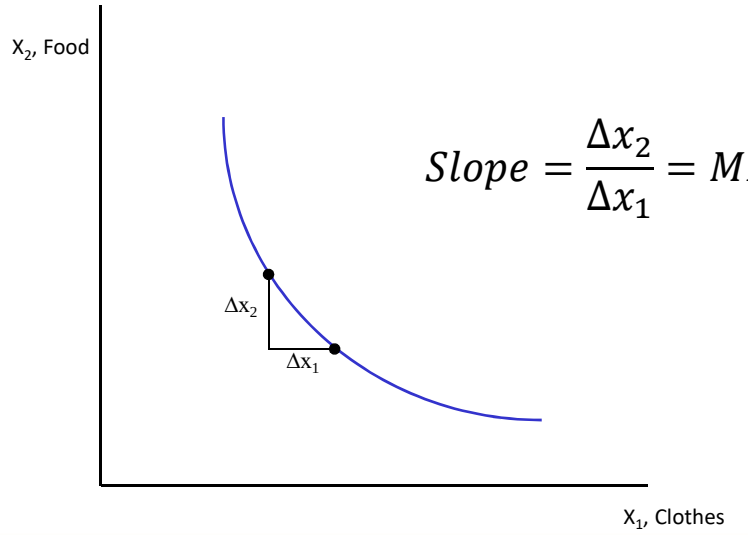
$$P_1X_1 + P_2X_2 \geq P_1Y_1 + P_2Y_2$$

AND NOT $\quad Q_1Y_1 + Q_2Y_2 \geq Q_1X_1 + Q_2X_2$

The Marginal Rate of Substitution I

- Joe consumes a given bundle, (x_1, x_2)
  - If we reduce his consumption of good 1, by Δx_1… how much good 2, Δx_2, is needed to put Joe back on the same indifference curve?
- The rate at which the consumer is just willing to substitute one good for the other: $\frac{\Delta x_2}{\Delta x_1}$
- With Δx_1 being a small change, we talk about the *marginal* rate of substitution (MRS)
  - This is the slope of an indifference curve at a particular point
  - It will be negative (which we also know from monotonicity)

The Marginal Rate of Substitution II



$$Slope = \frac{\Delta x_2}{\Delta x_1} = MRS$$

The Marginal Rate of Substitution III

$$MU_1 = \frac{\Delta U}{\Delta x_1} = \frac{u(x_1+\Delta x_1, x_2)-u(x_1, x_2)}{\Delta x_1}$$

$$\Delta U = MU_1 \Delta x_1$$

- To keep utility constant (ΔU = 0), if $x_1 \uparrow$, $x_2 \downarrow$

$$\Delta U = MU_1 \Delta x_1 + MU_2 \Delta x_2 = 0$$

- We know $MRS = \frac{\Delta x_2}{\Delta x_1}$ and so $MRS = \frac{MU_1}{MU_2}$

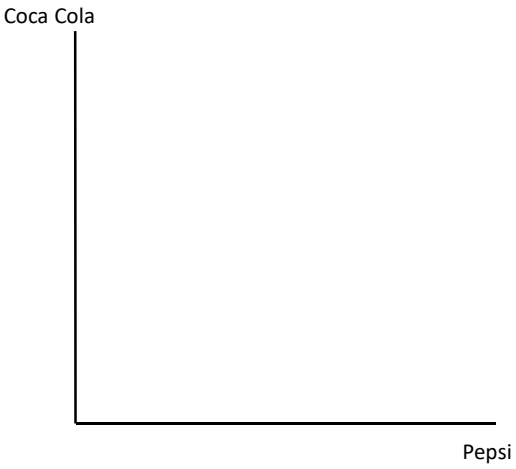
The Marginal Rate of Substitution IV

- We refer to small changes in x_1 and x_2 and so

$$du = \frac{\partial u(x_1, x_2)}{\partial x_1} dx_1 + \frac{\partial u(x_1, x_2)}{\partial x_2} dx_2 = 0$$

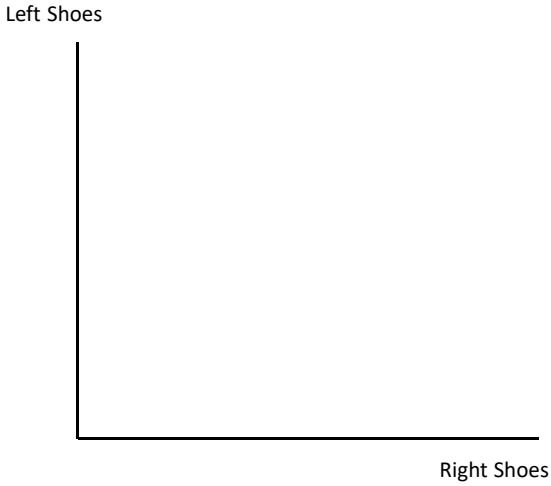
$$\frac{dx_2}{dx_1} = \frac{\frac{\partial u(x_1, x_2)}{\partial x_1}}{\frac{\partial u(x_1, x_2)}{\partial x_2}} = \text{MRS}$$

The MRS and Perfect Substitutes



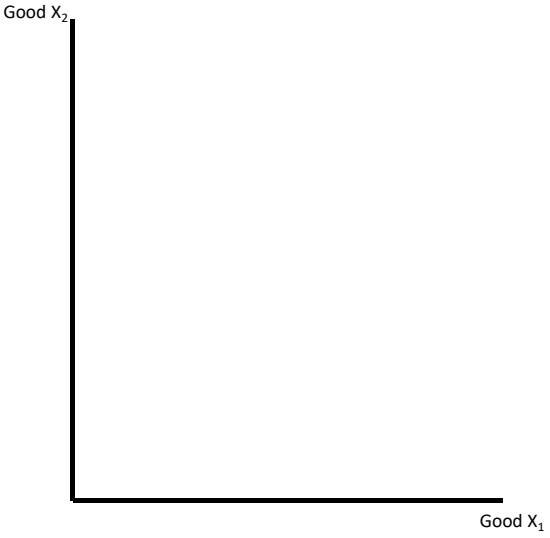
- $U(C, P) = aC + bP$
  - $MRS =$
- What happens to the MRS as we move down the indifference curve?

The MRS and Perfect Complements



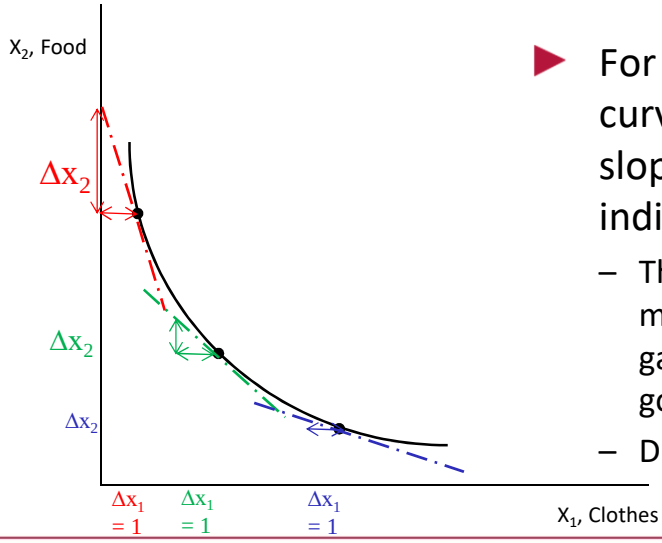
- $U(S_R, S_L) = min\{aS_R, bS_L\}$
- MRS is:

The MRS and Cobb-Douglas preferences



- $U(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$
- MRS is:
  - $\frac{MUx_1}{MUx_2} =$
- A monotonic transformation
  - $U(x_1, x_2) = tx_1^{\alpha} tx_2^{1-\alpha}$
  - $MRS =$
  - MRS does not depend on the utility representation

Diminishing Marginal Rate of Substitution



- For strictly convex indifference curves, what happens to the slope as we move down the indifference curve?
  - The more of a good you have, the more willing you are to sacrifice it to gain an additional unit of another good
  - Diminishing MRS (absolute value)

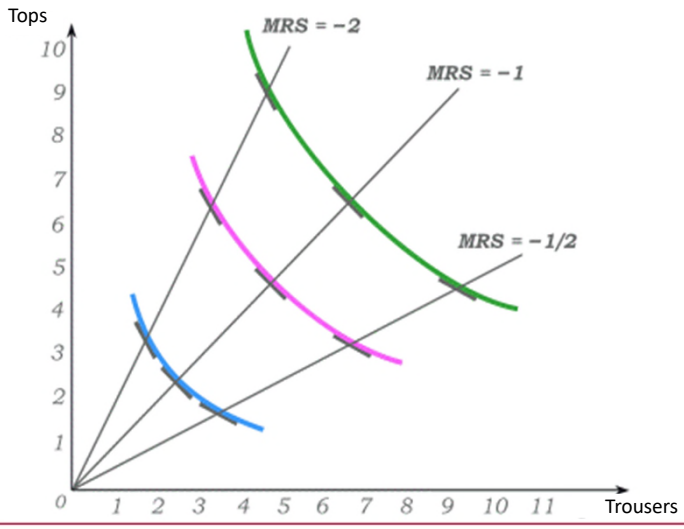
Homothetic tastes I

- A situation where the consumer's preferences depend solely on the ratio of good 1 to good 2
- Homothetic tastes give rise to indifference maps where the MRS is constant along any ray from the origin.

$$(x_1, x_2) \succ (y_1, y_2)$$
$$(tx_1, tx_2) \succ (ty_1, ty_2)$$

- When income rises, demand rises by the same proportion

Homothetic tastes II



- The relative quantity of each good remains constant along any ray from the origin
- When income is scaled up or down by t > 0, the demanded bundle scales up or down by the same amount.
  - The ratio of trousers to tops remains constant as income changes

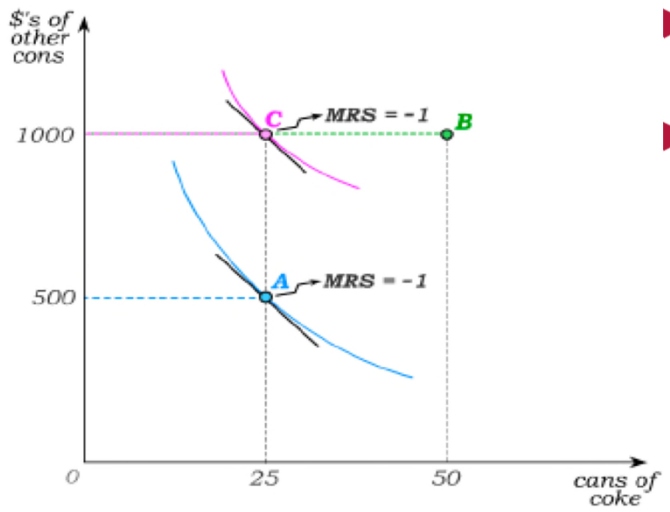
Quasilinear tastes

- Tastes are linear in one good, but may not be in the other good:

$$u(x_1, x_2) = v(x_1) + x_2$$

- With quasilinear tastes, indifference curves are vertical translates of one another.
- In this case, the MRS is constant along any vertical line from the x-axis.

Coke and all other consumption



- At A, MRS = -1: we will trade $1 for 1 can of coke.
- At B, will we value the 50th can of coke the same as the 25th?
  - More likely that the 25th can is valued the same regardless of how much in other consumption we undertake

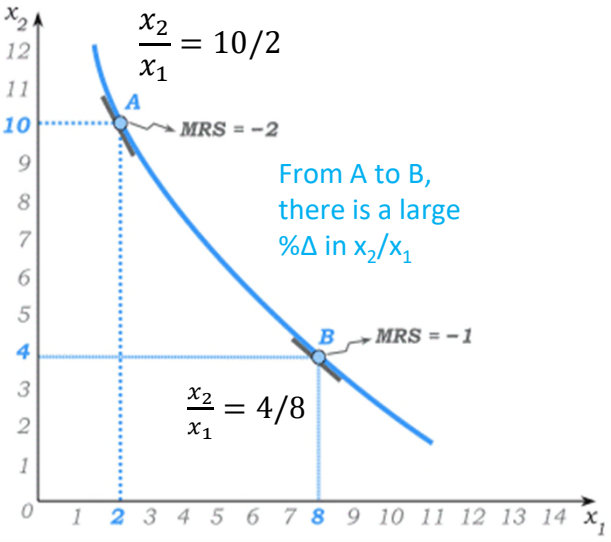
Elasticity of Substitution I

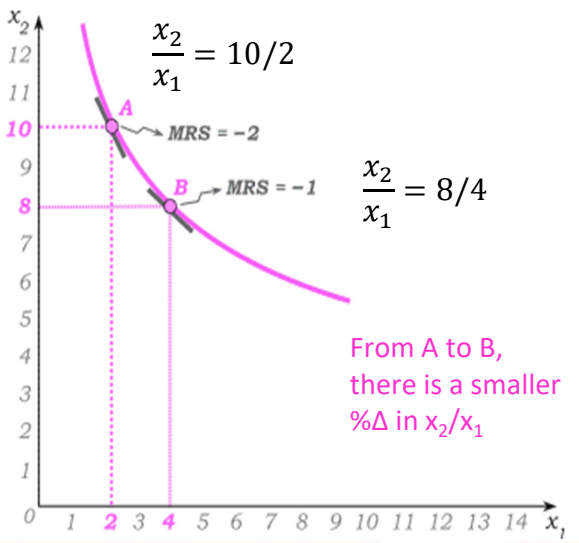
- How does the consumer substitute between x_1 and x_2?
- The degree of substitutability measures how responsive the bundle of goods along an IC is to changes in the MRS
- The elasticity of substitution is defined as:

$$\sigma = \left| \frac{\%\Delta(x_2/x_1)}{\%\Delta MRS} \right|.$$

  - Perfect substitutes: perfect substitutability: σ = ∞
  - Perfect complements: no substitutability: σ = 0
  - Cobb-Douglas: σ = 1

Elasticity of Substitution II





Elasticity of Substitution III

$$\sigma = \left| \frac{\%\Delta(x_2/x_1)}{\%\Delta MRS} \right|.$$

- Consider a 1% change in the MRS
- The less curved the IC, the more x_2 has to fall and the more x_1 has to increase for the MRS to have changed by 1%
- Thus, the less curvature in the IC, the greater is the % $\Delta x_2/x_1$ required for the MRS to change by 1%, which implies a higher elasticity of substitution σ
  - i.e. Perfect Substitutes

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OPTIMISATION

Optimisation I

Consumers choose the best bundle within their feasible set

Remember

Slope of budget constraint: $\frac{Px_1}{Px_2}$

=

Slope of indifference curve:

$\text{MRS} = \frac{MUx_1}{MUx_2}$

Optimisation II

- Consumers maximise utility subject to a budget constraint
- This occurs at a tangency, where

$$MRS = \frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} = \frac{MUx_1}{MUx_2} = \frac{Px_1}{Px_2}$$

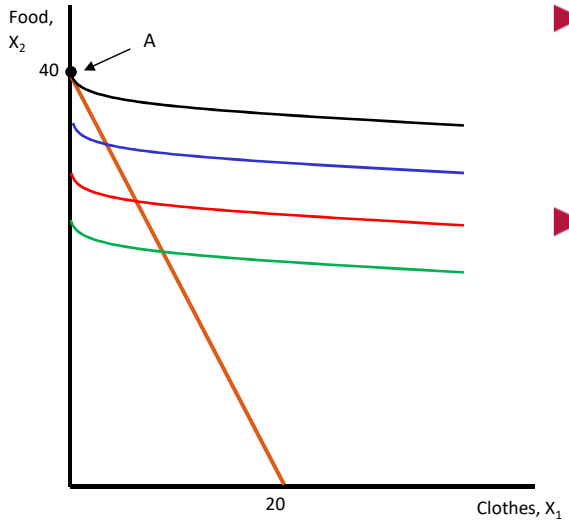
- Why must this hold for an interior solution?
  - Assume two goods X and Y
  - What would happen if: $\frac{MU_X}{MU_Y} \neq \frac{P_X}{P_Y}$?

Optimisation III

Say: $MRS = \frac{\Delta Y}{\Delta X} = \frac{MU_X}{MU_Y} = \frac{1}{1} > \frac{P_X}{P_Y} = \frac{2}{3}$

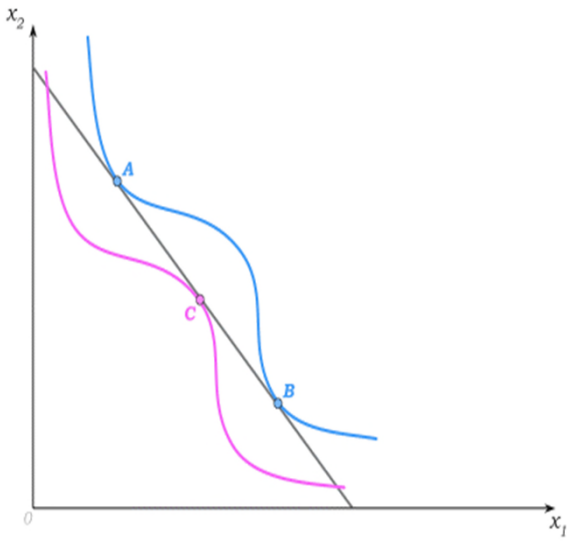
- To consume 1 more unit of X, Joe is willing to give up 1Y (MRS)
- To consume 1 more unit of X, the market only requires Joe to give up $\frac{2}{3}$ Y (slope of budget constraint)
- If Joe is willing to give up 1 unit of Y, he'll certainly give up only $\frac{2}{3}$ Y
- Consumption of X will rise and Y will fall until: $\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$

Optimisation IV: Corner Solutions



- If the optimal choice involves consuming both goods, then the tangency condition must hold
  - A necessary condition
- What happens at A?

Optimisation V: non-convexity



- The tangency condition is necessary for optimality, but it is not sufficient unless preferences are convex
- If we have a tangency, we don't always have an optimal choice
  - Points A, B and C are all interior tangency conditions

Optimising mathematically: The Primal

- If we know income, prices and the utility function:

$$\max_{x_1,x_2,\lambda} L = U(x_1, x_2) + \lambda(M - P_1x_1 - P_2x_2)$$

we can solve for the Marshallian demands: $x^*(p_1, p_2, M)$

  - Demand is homogenous of degree zero
    - $x^*(p_1, p_2, M) = x^*(kp_1, kp_2, kM) \quad for\ k > 0$
  - If utility is monotonic then the budget binds
- Tangency is necessary, not sufficient. Only sufficient if preferences are convex
  - If preferences are not convex, check SOC to ensure a maximum

The Lagrange multiplier

- Tangency implies:
- $\lambda^* = \frac{MU_1}{P_1} = \frac{MU_2}{P_2} = \cdots = \frac{MU_i}{P_i} = \cdots = \frac{MU_n}{P_n}$
- λ^* is the marginal utility of an extra £ of expenditure
  - The marginal utility of income
  - £1 of extra income will increase utility by λ
- Price is the consumer's evaluation of the utility of the last unit consumed
- $P_i = \frac{MU_i}{\lambda}$ for every i

Marshallian demands I

- Perfect substitutes:

$$x_1^* = \begin{cases} M/P_1 & if\ P_1 < P_2 \\ any\ number\ between\ 0\ and\ M/P_1 & if\ P_1 = P_2 \\ 0 & if P_1 > P_2 \end{cases}$$

- Perfect complements:
  - $x_1^* = M/(P_1 + P_2)$
- Neutrals and bads:
  - Spend money only on the good you like (x_1)
  - $x_1^* = M/P_1$ and $x_2^* = 0$

Marshallian demands II

- Cobb-Douglas: $u = x_1^c x_2^d$

  $x_1^* = \frac{c}{c+d}\frac{M}{P_1}$ and $x_2^* = \frac{d}{c+d}\frac{M}{P_2}$

- It is convenient to write Cobb-Douglas utility functions with exponents that sum to 1
  - Raise u to power $1/(c+d)$ $\gg$ $v = x_1^{\alpha} x_2^{1-\alpha}$
- A fixed proportion of income is spent on each good
  - The size is determined by the exponents, e.g. good 1 = α
  - Marshallian demand: $x_1^* = \frac{\alpha M}{P_1}$ and $x_2^* = \frac{(1-\alpha)M}{P_2}$

An example

$u\left(x_1^{0.5}, x_2^{0.5}\right);\ P_1 = 20;\ P_2 = 10; M = 200$

$L = x_1^{0.5} x_2^{0.5} + \lambda(M - P_1 x_1 - P_2 x_2)$

FOCs

$$\frac{\partial L}{\partial x_1} = 0.5 x_1^{-0.5} x_2^{0.5} - \lambda P_1 = 0$$

$$\frac{\partial L}{\partial x_2} = 0.5 x_1^{0.5} x_2^{-0.5} - \lambda P_2 = 0$$

$$\Rightarrow \quad \frac{x_2}{x_1} = \frac{P_1}{P_2} \quad >> \quad P_1 x_1 = P_2 x_2$$

$$\frac{\partial L}{\partial \lambda} = M - P_1 x_1 - P_2 x_2 = 0 \quad >> \quad M = 2P_1 x_1 \quad \gg \quad x_1^* = \frac{M}{2P_1}$$

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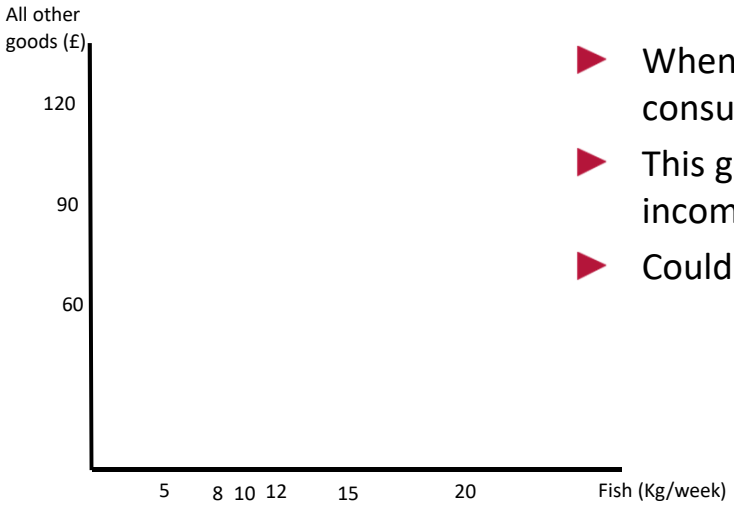
COMPARATIVE STATICS

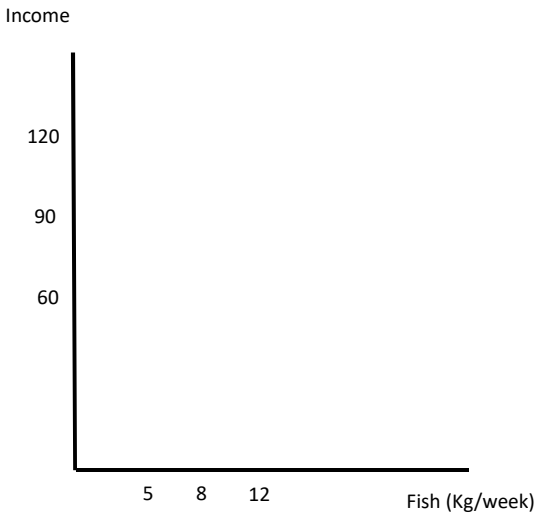
Changing optimal consumption I



- When income rises, optimal consumption moves: A – B – C.
- This gives the income-offer or income-consumption curve
- Could this curve's shape vary?

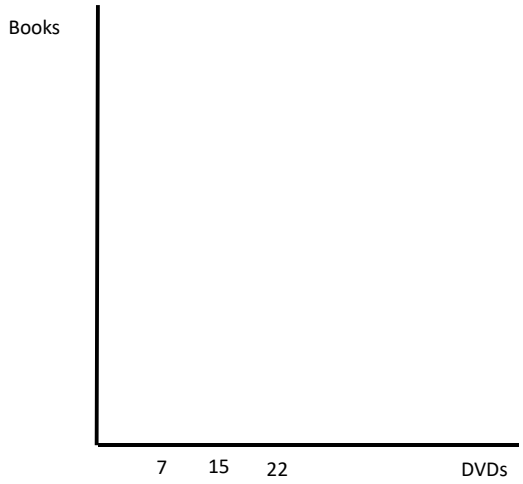
Income	Quantity demanded
120	12
90	8
60	5

The Engel curve



- We derive the Engel curve from the income-offer curve
  - Holding prices constant, how does quantity demanded vary with income?
- For a normal good, as income rises, quantity demanded rises
- For an inferior good, as income rises, quantity demanded falls
  - The Engel curve may not slope upwards

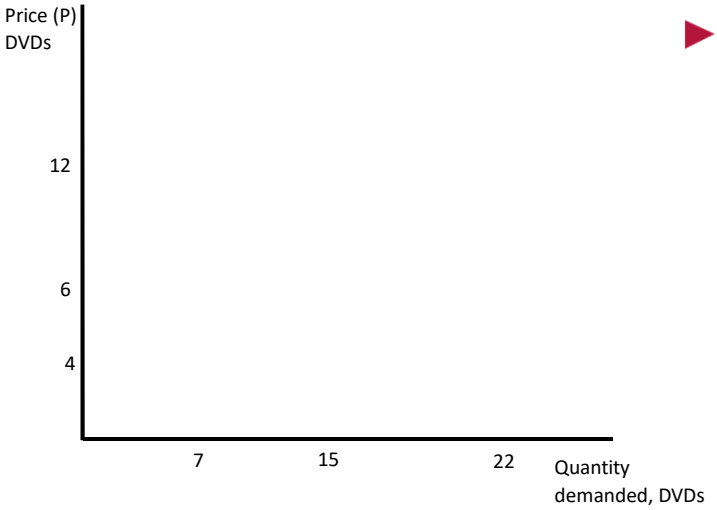
Changing optimal consumption II



- When price falls, optimal consumption moves: A – B – C.
- This gives the price-offer or price-consumption curve

Price of DVDs	Quantity demanded
12	7
6	15
4	22

The Demand Curve I



- We derive the demand curve for DVDs from the price-offer curve
  - Holding income, other prices and preferences constant, how does quantity demanded change following a price change?
  - What happens to demand if income, other prices or preferences change?

The Demand Curve II

- The law of demand
- The *substitution effect:* As $P_x \downarrow$, good Y becomes relatively more expensive and less attractive to the consumer.
- Even remaining on the same IC, the price ratio changes
  - Optimal demands will change to where: MRS = NEW price ratio
- The *income effect:* As $P_x \downarrow$, real income rises. The consumer now feels richer, so quantity demanded changes.
  - As real income changes, the consumer must move to a new IC

Marshallian demand elasticities I

- When price falls, quantity demanded rises
  - By how much?
- The price elasticity of demand ε_{x,p_x} measures the percentage change in quantity demanded in response to a percentage change in a good's own price.

$$\varepsilon_{x,p_x} = \frac{\Delta x/x}{\Delta p_x/p_x} = \frac{\Delta x}{\Delta p_x}\frac{p_x}{x} = \frac{\partial x}{\partial p_x}\frac{p_x}{x}$$

- What is the sign of price elasticity of demand?

Marshallian demand elasticities II

- Income elasticity of demand $\xi_{x,M}$ measures the percentage change in quantity demanded in response to a percentage change in income

$$\xi_{x,M} = \frac{\Delta x/x}{\Delta M/M} = \frac{\Delta x}{\Delta M}\frac{\mathrm{M}}{x} = \frac{\partial x}{\partial M}\frac{M}{x}$$

- Cross price elasticity of demand, e_{x,p_y} measures the percentage change in quantity demanded in response to a percentage change in the price of another good

$$e_{x,p_y} = \frac{\Delta x/x}{\Delta p_y/p_y} = \frac{\Delta x}{\Delta p_y}\frac{p_y}{x} = \frac{\partial x}{\partial p_y}\frac{p_y}{x}$$

Income and substitution effects

- Two goods: apples and bananas and price of apples falls
- Apples are relatively cheaper (substitution effect), so the consumer can afford to buy more (income effect)
  - How many more are bought just because they are cheaper?
  - What's the change in demand ONLY due to the substitution effect?
- Graphical analysis: what we do
  - When price falls (BC pivots), real income rises … but ignore that by
  - … taking some income from the consumer (shift new BC) so …
  - … utility is the same after the price change as it was before (new BC tangent to old IC)

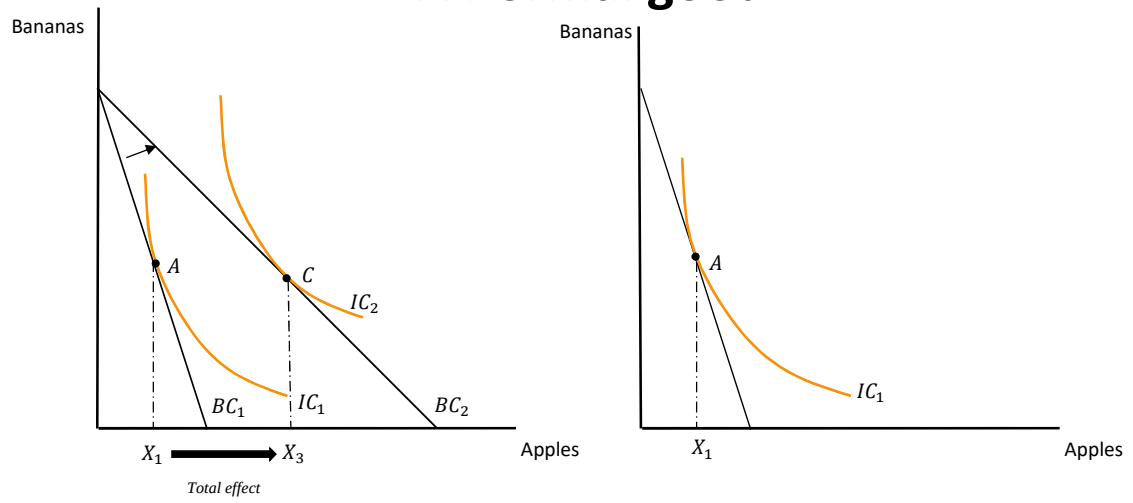
Expenditure minimisation

- For the substitution effect, we want the consumer to receive a given level of utility, say v, at lowest cost
  - We minimise expenditure subject to a given level of utility
- Choose $x_1, x_2, \ldots, x_n$ to solve the following problem:

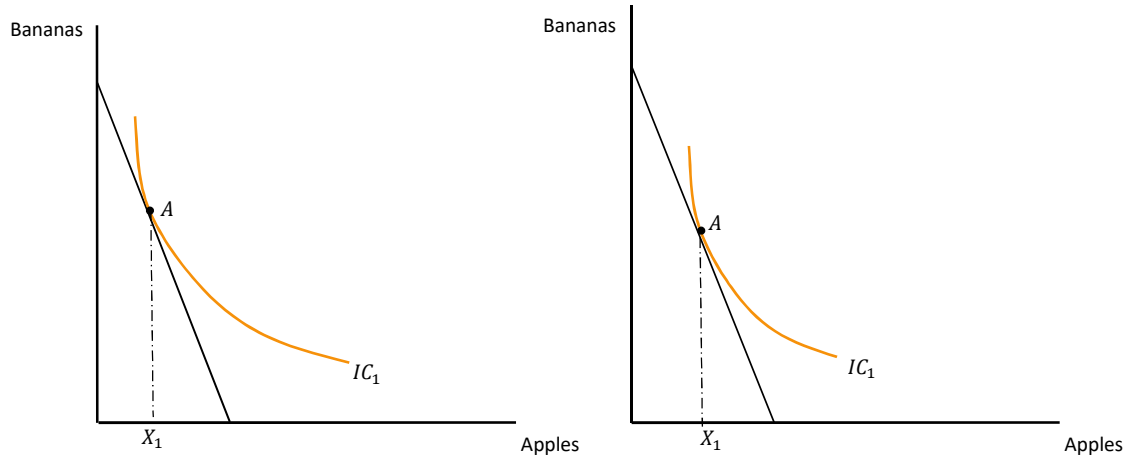
$$min\, p_1x_1, p_2x_2 \ldots p_nx_n \qquad s.t. \quad U(x_1, x_2, \ldots, x_n) \geq v$$

- The solution to the problem gives the Hicksian demands of the form: $H_1^*(p_1, p_2, v)$
  - These are also known as the compensated demands

The Hicks income and substitution effects: A normal good



The Hicks income and substitution effects:



Inferior Good

Giffen Good

The Dual Problem I

- Find the initial bundle (at original price ratio)
  - Utility maximisation: Consume on highest IC subject to BC
- Find the change in demand due to the substitution effect
  - Expenditure minimisation: what is the least costly way of achieving the original level of utility at the new price ratio?
- Find the new bundle (at new price ratio)
  - Utility maximisation (now also takes into account the income effect)

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The Dual Problem II

$u(x_1, x_2) = x_1^{0.5} x_2^{0.5} \quad P_1 = 20; P_2 = 10; M = 200. \quad P_1 \downarrow to\ 10$

Step 1

$$\max u(x_1, x_2) = x_1^{0.5} x_2^{0.5} \ \ subject\ to\ P_1 x_1 + P_2 x_2 \leq M$$

$x_1^* = \frac{M}{2P_1} = \frac{200}{(2\times 20)} = 5$ and $x_2^* = \frac{M}{2P_2} = \frac{200}{(2\times 10)} = 10$

Step 2

- Find the level of utility at the optimal bundle:
- $u(x_1, x_2) = x_1^{0.5} x_2^{0.5} = 5^{0.5} 10^{0.5} = 7.071 \ldots$

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The Dual Problem III

Step 3: find the compensated budget

$$\min \exp = 10x_1 + 10x_2 \ \ subject\ to\ x_1^{0.5} x_2^{0.5} = 7.071$$

$$L = 10x_1 + 10x_2 + \lambda(7.071 - x_1^{0.5} x_2^{0.5})$$

$$\frac{\partial L}{\partial x_1} = 10 - 0.5\lambda x_1^{-0.5} x_2^{0.5} = 0$$

$$\frac{\partial L}{\partial x_2} = 10 - 0.5\lambda x_1^{0.5} x_2^{-0.5} = 0$$

$$\frac{\partial L}{\partial \lambda} = 7.071 - x_1^{0.5} x_2^{0.5} = 0$$

$\frac{x_2}{x_1} = 1 \quad \gg \quad x_2 = x_1$

$x_1 = x_2 = 7.071$

Substitution effect: x_1 rises from 5 to 7.071

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The Dual Problem IV

Step 4

$$\max u(x_1, x_2) = x_1^{0.5} x_2^{0.5} \ \ subject\ to\ P_1 x_1 + P_2 x_2 \leq M$$

$$\frac{\partial L}{\partial x_1} = 0.5 x_1^{-0.5} x_2^{0.5} - \lambda P_1 = 0$$

$$\frac{\partial L}{\partial x_2} = 0.5 x_1^{0.5} x_2^{-0.5} - \lambda P_2 = 0$$

$$\frac{x_2}{x_1} = \frac{P_1}{P_2} \quad \gg \quad \frac{x_2}{x_1} = 1$$

$$x_1 = x_2$$

$$P_1 x_1 + P_2 x_1 = M \quad \gg \quad x_1^* = \frac{M}{(P_1 + P_2)} = \frac{200}{(10 + 10)} = 10$$

Income effect: x_1 rises from 7.071 to 10

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Connecting the results I

- In both cases, Ps and M are given and we choose optimal x_i
- The solutions to the primal and dual must be consistent
- Utility maximisation yields the Marshallian demands
  - $x_1^* = x_1^*(p_1, p_2, M)$
  - $x_2^* = x_2^*(p_1, p_2, M)$
- We can use these ordinary demands to find the Indirect Utility function
- And to derive Roy's Identity

Indirect Utility Function

- Once we know the Marshallian demands; x_1^*, x_2^* we can find the actual level of utility:

$$v(p_1, p_2, M) = U(x_1^*(p_1, p_2, M), x_2^*(p_1, p_2, M))$$

- This is called the indirect utility function, where optimal level of utility depends indirectly on prices and income
- It has the following properties:
  - It is non-increasing in every price, decreasing in at least one price
  - Increasing in Income
  - Homogeneous of degree zero in price and income

Roy's Identity I

- Marshallian demands found from indirect utility function
- Use the envelope theorem to find:
- $\frac{\partial V}{\partial P_1} = \lambda x_1^*$ (Marginal disutility of p_1)
- $\frac{\partial V}{\partial M} = \lambda$ (Marginal utility of income)
- Simple arithmetic gives Roy's Identity:

$$\frac{\partial V / \partial P_1}{\partial V / \partial M} = x_1^*(P_1, P_2, M)$$

Roy's Identity II

$$L = U(x_1 x_2) + \lambda(M - P_1 x_1 - P_2 x_2)$$

- The numerator:

$$\frac{\partial V(P_1,P_2,M)}{\partial P_i} = \frac{\partial u(x_1,x_2)}{\partial P_i} + \lambda\left(\frac{\partial (M - P_1 x_1 - P_2 x_2)}{\partial P_i}\right)$$

- But $u(x_1 x_2)$ does not depend on P_i
- $\frac{\partial u(x_1,x_2)}{\partial P_i} = 0$
- So $\frac{\partial V(P_1,P_2,M)}{\partial P_i} = -\lambda x_i(P_1, P_2, M)$

Roy's Identity III

$$L = U(x_1 x_2) + \lambda(M - P_1 x_1 - P_2 x_2)$$

- The denominator

$$\frac{\partial V(P_1,P_2,M)}{\partial M} = \frac{\partial u(x_1,x_2)}{\partial M} + \lambda\left(\frac{\partial (M - P_1 x_1 - P_2 x_2)}{\partial M}\right)$$

- But $u(x_1 x_2)$ does not depend on M
- $\frac{\partial u(x_1,x_2)}{\partial M} = 0$ >> $\frac{\partial V(P_1,P_2,M)}{\partial M} = \lambda$
- So: $\frac{\partial V/\partial P_1}{\partial V/\partial M} = \frac{-\lambda x_i(P_1,P_2,M)}{\lambda} = x_1^*(P_1, P_2, M)$

Connecting the results II

- Utility maximisation gives Marshallian demands, indirect Utility and Roy's Identity
- Expenditure minimisation yields the Hicksian demands
  - $x_1^* = h_1^*\ (p_1, p_2, v)$
  - $x_2^* = h_2^*\ (p_1, p_2, v)$
- We can use these Hicksian demands to find the Expenditure function
- And to derive Shephard's Lemma

Expenditure Function

- Once we know the Hicksian demands; h_1^*, h_2^* we can find the minimum expenditure needed to achieve given utility v:
  $$M = E(p_1, p_2, v) = p_1 H_1^*(p_1, p_2, v) + p_2 H_2^*(p_1, p_2, v)$$
- This is called the Expenditure function, which maps prices and utility to minimal expenditure
- It has the following properties:
  - It is non-decreasing in every price, increasing in at least one price
  - Increasing in utility
  - Homogeneous of degree 1 in all prices p

Shephard's Lemma I

- Compensated demands can be found from expenditure function
- Use the envelope theorem :
- $\frac{\partial E}{\partial P_i} = \frac{\partial L}{\partial P_i} = h_i(P_1, P_2, v)$
- Shephard's Lemma:

$$\frac{\partial E(P_1, P_2, v)}{\partial P_1} = h_1^*(P_1, P_2, v)$$

Shephard's Lemma II

$$L = P_1 x_1 + P_2 x_2 + \lambda(v - u(x_1, x_2))$$

$$\frac{\partial E(P_1,P_2,v)}{\partial P_i} = \frac{\partial (P_1 x_1 + P_2 x_2)}{\partial P_i} + \lambda\left(\frac{\partial (v - u(x_1,x_2))}{\partial P_i}\right)$$

- But $[v - u(x_1, x_2)]$ does not depend on P_i
- $\lambda\left(\frac{\partial (v - u(x_1,x_2))}{\partial P_i}\right) = 0$
- So $\frac{\partial E(P_1,P_2,v)}{\partial P_i} = x_i = h_i(P_1, P_2, v)$

Connecting the results III

- Look at the symmetry

$$L = u(x_1 x_2) + \lambda\left(M - \sum_{i=1}^{n} p_i x_i\right)$$

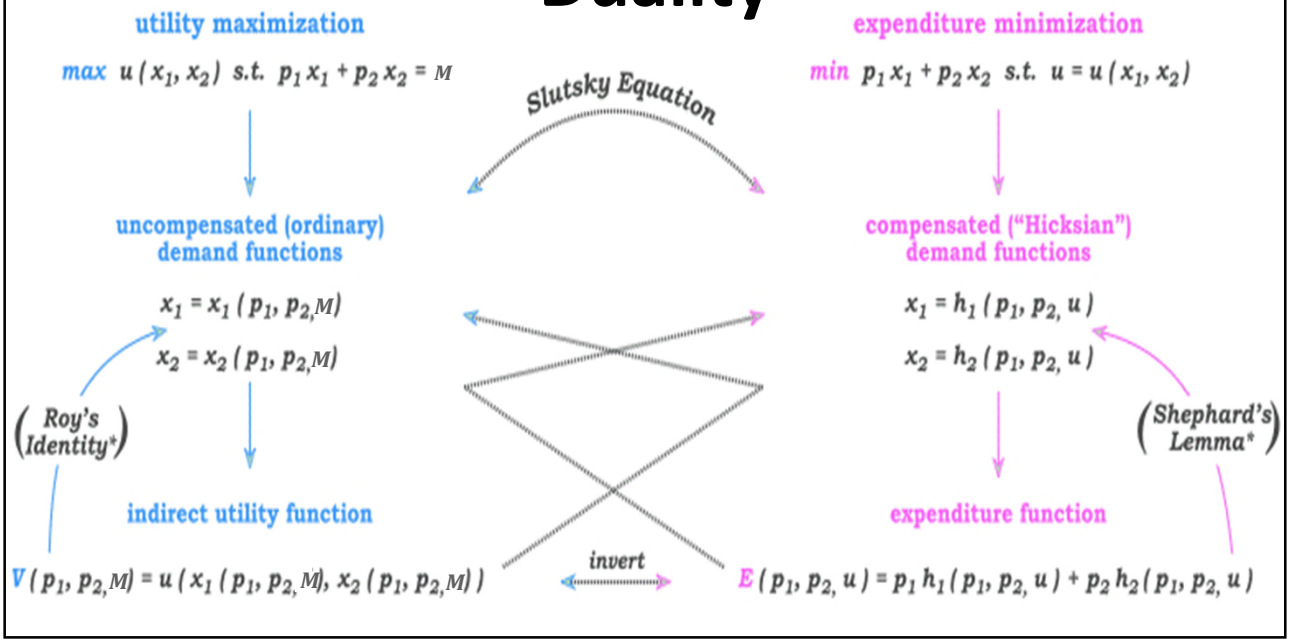
$$L = \sum_{i=1}^{n} p_i x_i + \lambda(v - u(x_1 x_2))$$

- Constraint in primal becomes objective in dual

$$v = V(p_i, M) = V(p_i, E(p_i, v))$$

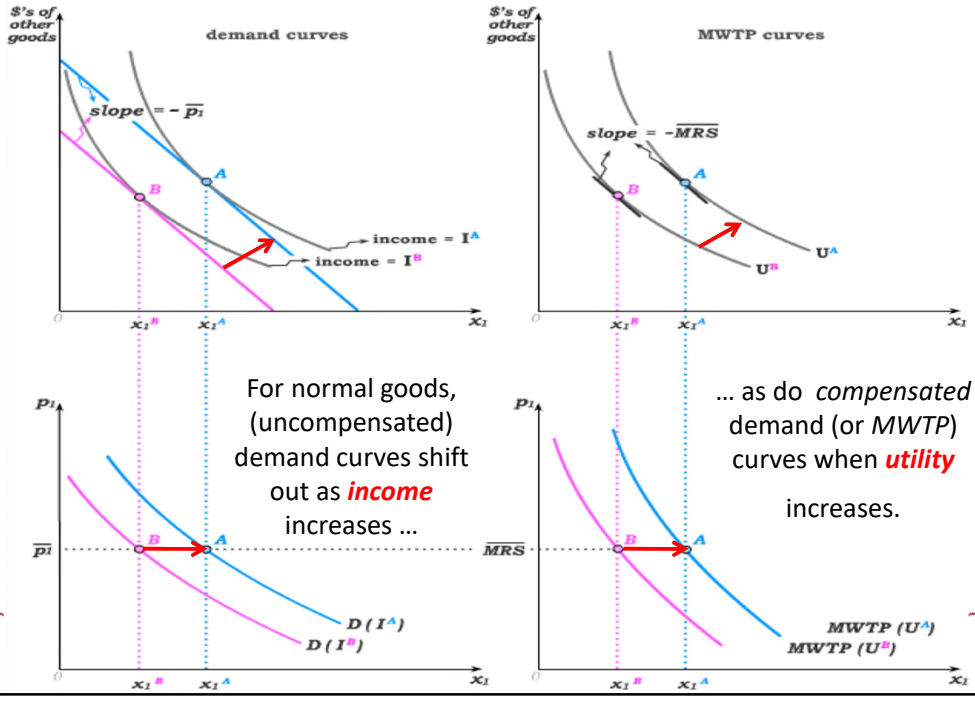
$$M = E(p_i, v) = E(p_i, V(p_i, M))$$

Duality



Compensated and Uncompensated Demand

- Marshallian demand or uncompensated demand curve
  - As P_x falls, consumer moves to higher IC; real income rises, while nominal income is held constant
  - This curve accounts for both the income and substitution effects
- The compensated or Hicksian demand curve
  - As P_x falls, hold real income (or utility) constant; consumer is 'compensated' and forced to remain on same Indifference curve
  - This curve accounts for only the substitution effect
  - Actual level of utility varies along the demand curve
- Consider demand curves for normal, inferior, Giffen goods



For normal goods, (uncompensated) demand curves shift out as ***income*** increases …

… as do *compensated* demand (or *MWTP*) curves when ***utility*** increases.

The reverse is true for inferior goods …

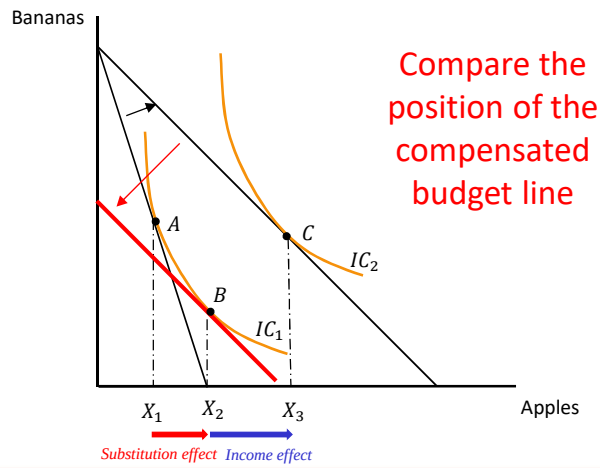
… and no shifts in these curves occurs when the good is quasilinear.

Slutsky Substitution

- The Hicks substitution compensates the consumer so that he can remain on the same indifference curve
  - It keeps utility constant (following the price change)
- The Slutsky substitution compensates the consumer so that he can still consume his original bundle
  - It keeps purchasing power constant (following the price change)
- The process is the same as the Hicks substitution effect, but now the compensated budget line won't be tangential to the original indifference curve…
- It will pass through the original consumption bundle

Slutsky versus Hicks: Normal good

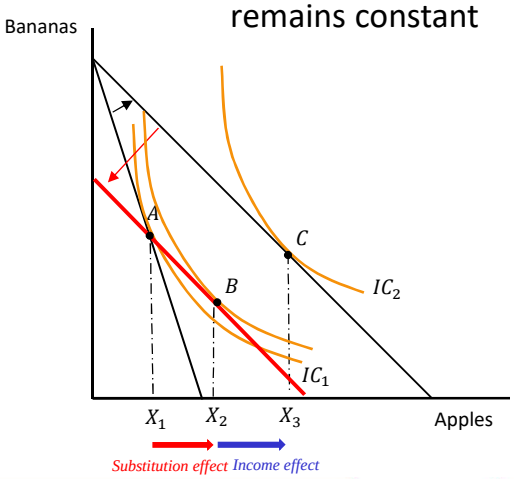
Utility remains constant



Compare the position of the compensated budget line

Hicksian

Purchasing power remains constant



Slutsky

A familiar example

$u(x_1, x_2) = x_1^{0.5} x_2^{0.5}$ $P_1 = 20; P_2 = 10; M = 200.$ $P_1 \downarrow to\ 10$

- We already know initial bundle is: $x_1^* = 5$ and $x_2^* = 10$
- Hicksian demands were: $x_1^H = x_2^H = 7.071$
- New bundle: $x_1^M = x_2^M = 10$
- But, Hicksian demands compensated consumer such that his utility was unaffected by price change
- Slutsky compensates the consumer such that at the new price ($P_1 = 10$), he can still consume: $x_1^* = 5$ and $x_2^* = 10$
  - To consume this bundle, by how much must income change?

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Slutsky equation I

- Find the change in income needed to make the original bundle (5, 10) affordable at new price, P_1'

Original Income: $M = P_1 x_1 + P_2 x_2$

New Income: $M' = P_1' x_1 + P_2 x_2$

$\Delta M = M' - M = P_1' x_1 - P_1 x_1 = x_1 \Delta P_1 = 5(10 - 20) = -50$

- Income must fall by 50, from 200 to 150
- This gives consumer just enough income to purchase the original bundle at the new price ratio

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Slutsky equation II

$$L = x_1^{0.5} x_2^{0.5} + (150 - P_1 x_1 - P_2 x_2)$$

$$x_1 = \frac{M}{(P_1 + P_2)} = \frac{150}{(10 + 10)} = 7.5 = x_2$$

- Under Hicks substitution, x_1 increases from 5 to 7.071
  - Income effect then causes x_1 to rise from 7.071 to 10
- Under Slutsky substitution, x_1 increases from 5 to 7.5
  - Income effect then causes x_1 to rise from 7.5 to 10

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Slutsky equation III

Substitution effect: $\Delta x_1^s = x_1(P_1', M') - x_1(P_1, M)$

Income effect: $\Delta x_1^n = x_1(P_1', M) - x_1(P_1', M')$

Total effect: $\Delta x_1 = x_1(P_1', M) - x_1(P_1, M) = \Delta x_1^s + \Delta x_1^n$

- Express as rates of change by defining Δx_1^m as $-\Delta x_1^n$
- $\Delta x_1 = \Delta x_1^s - \Delta x_1^m$
- Divide each side by ΔP_1 $\quad \frac{\Delta x_1}{\Delta P_1} = \frac{x_1^s}{\Delta P_1} - \frac{\Delta x_1^m}{\Delta P_1}$

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Slutsky equation IV

$$\frac{\Delta x_1}{\Delta P_1} = \frac{x_1^S}{\Delta P_1} - \frac{\Delta x_1^m}{\Delta P_1}$$ and recall: $\Delta M = x_1 \Delta P_1$ >> $\Delta P_1 = \frac{\Delta M}{x_1}$

Replace denominator in term 3 $$\frac{\Delta x_1}{\Delta P_1} = \frac{x_1^S}{\Delta P_1} - \frac{\Delta x_1^m}{\Delta M} x_1$$

1. Rate of change of x_1 following a change in P_1, holding M fixed (total effect)
2. Rate of change of x_1 as P_1 changes, adjusting M to keep old bundle affordable (substitution effect)
3. Rate of change of x_1, holding prices fixed and adjusting M (income effect)

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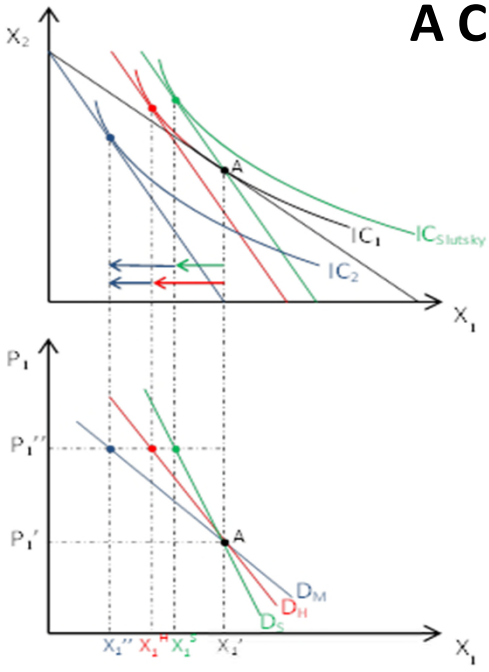
A Comparison: A Normal Good



D_M = Marshallian or own price demand curve

D_H = Hicksian Demand Curve (keeps utility constant)

D_S = Slutsky Demand Curve (keep purchasing power constant)

WARWICK

WELFARE

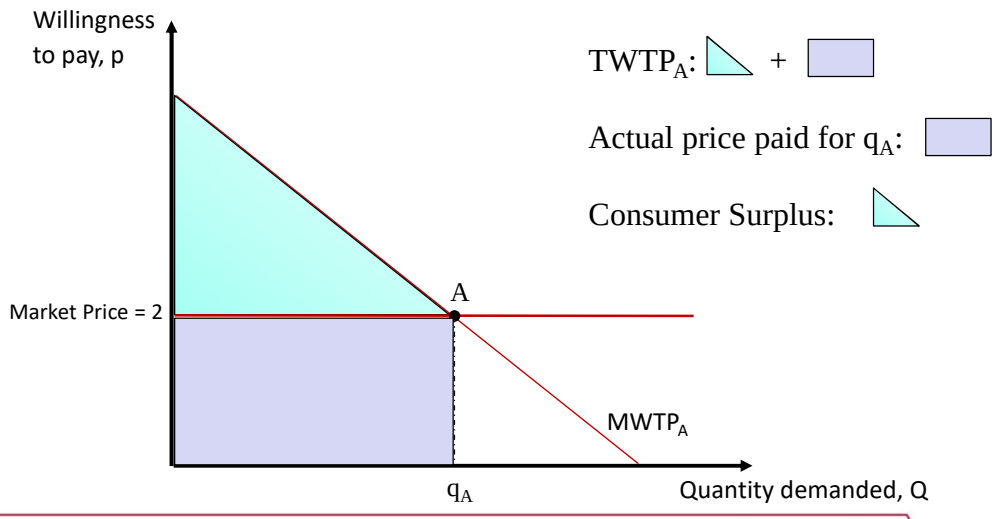
A change in welfare I

- In policy debates it is important to be able to ***quantify*** how consumer "welfare" is affected by changing prices.
  - How is welfare affected if fuel tax rises, or a carbon tax is introduced?
  - A key part of economists' role in government, regulators, consulting
- We can't look at utility directly (ordinal utility), so we use a proxy – income or money
- There are 3 ways that changes in welfare can be measured
  - Consumer Surplus
  - Compensating variation
  - Equivalent variation

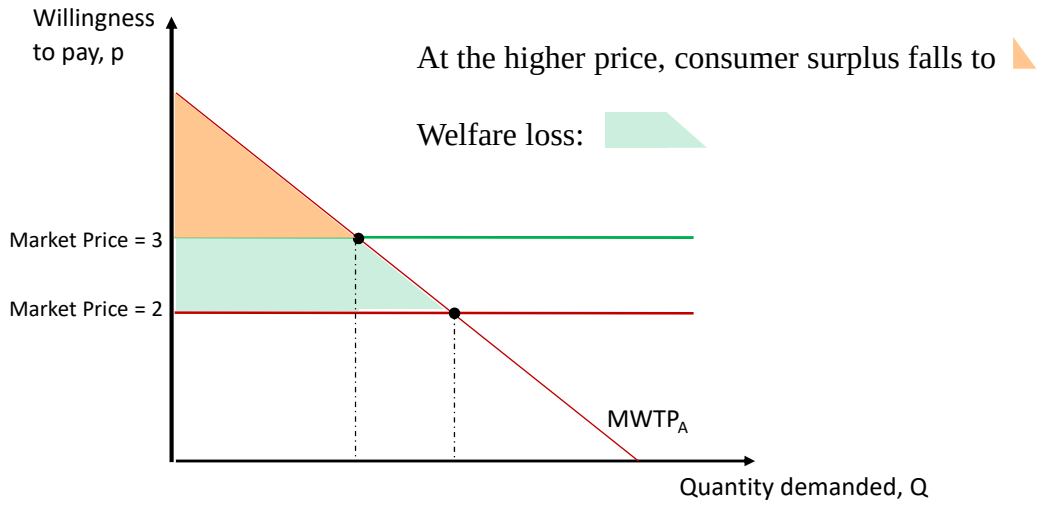
Consumer Surplus I

- We use compensated demand (MWTP) to measure it
- MWTP tells us how much each unit of a good is valued *given we are consuming at some bundle* ***A***.
- TWTP for all q^A *units* is: $\sum_{i=1}^{A} MWTP$
  - The area under the MWTP curve
- The difference between the TWTP and the actual amount paid gives the consumer surplus
- When price changes, we measure the change in welfare by measuring the change in consumer surplus

Consumer surplus II



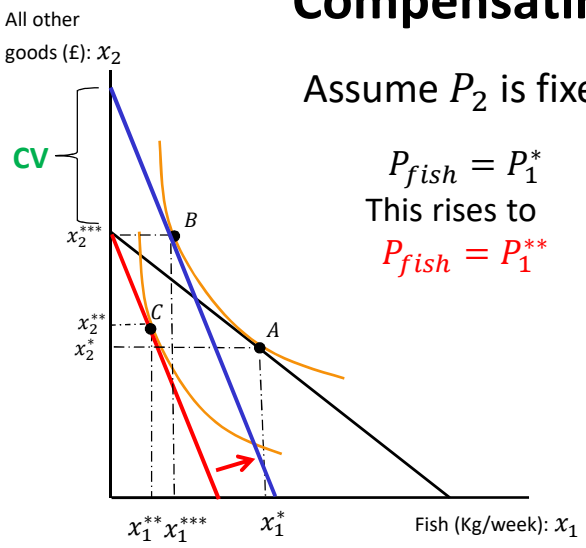
Consumer surplus III



Compensating Variation I

- Say the price of a good rises …
- How much money would the government have to give the consumer **after** the price change to make him just as well off as he was **before** the price change?
  - You should recognise this idea!
- How far should we shift the **new** budget line so that it is just tangential to the **original** indifference curve?

Compensating Variation II



Assume P_2 is fixed

$P_{fish} = P_1^*$
This rises to
$P_{fish} = P_1^{**}$

$$E_1(P_1^*, P_2, U_0) = P_1^* x_1^* + P_2 x_2^*$$

$$= P_1^{**} x_1^{**} + P_2 x_2^{**}$$

$$E_2(P_1^{**}, P_2, U_0) = P_1^{**} x_1^{***} + P_2 x_2^{***}$$

$$CV = E_2 - E_1$$

Compensating Variation III

- Shephard's Lemma: derivative of expenditure function with respect to P_x is the compensated demand function

$$H(P_1, P_2, v) = \frac{\partial E(P_1, P_2, v)}{\partial P_1}$$

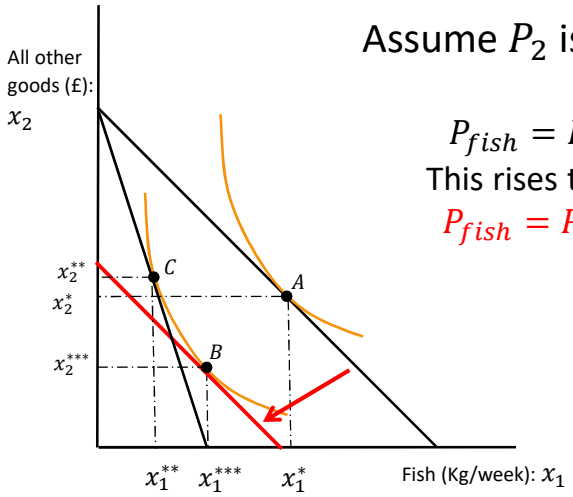
$$CV = E_2(P_1^{**}, P_2, U_0) - E_1(P_1^*, P_2, U_0)$$

- CV is the integral of the Hicksian demand
- This integral is the area to the left of the Hicksian demand curve between $P_1^{**} > P_1^*$

Equivalent Variation I

- Say the price of a good rises …
- How much money would have to be taken away from the consumer **before** the price change to make him just as well off as he would be **after** the price change?
  - What is the maximum amount you are willing to pay to avoid the price change?
- How far must we shift the **original** budget line so that it is just tangential to the **new** indifference curve?

Equivalent Variation II



Assume P_2 is fixed

$P_{fish} = P_1^*$
This rises to
$P_{fish} = P_1^{**}$

$$M_1 = P_1^* x_1^* + P_2 x_2^*$$

$$= P_1^{**} x_1^{**} + P_2 x_2^{**}$$

$$M_2 = P_1^* x_1^{***} + P_2 x_2^{***}$$

$$EV = M_1 - M_2$$

A change in welfare II

- Consumer surplus, compensating variation and equivalent variation can give different values
  - The same change in price can lead to different changes in welfare, depending on how we measure it
- £1 is worth differing amounts at different prices
- CS, CV and EV will only be equal to each other if tastes are quasilinear
  - As here, there is no income effect

WARWICK

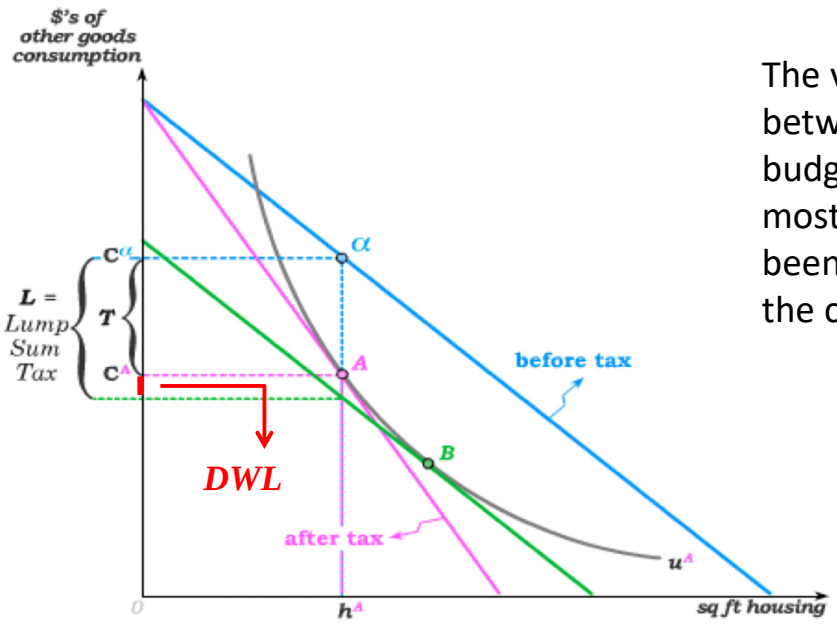
APPLICATIONS

The effect of a tax I

- Suppose the government imposes a specific tax on a good
- This alters relative prices and consumers, workers and savers substitute away from the taxed good
  - Specific taxes change opportunity costs and change our consumption plans, so we are worse off without contributing to the government's effort to raise revenues
- Therefore society incurs a deadweight loss
  - The size is related to the degree of substitutability between taxed and non taxed goods.

The effect of a tax II

- We quantify a DWL from a tax as the difference between the tax revenue that is raised and the tax revenue that could have been raised without making anyone worse off
- To see how much revenue **T** is raised, we need to know what consumers do after the tax is imposed
- We can compare this with how much income we could have taken from the consumer without distorting prices and without making the consumer worse off

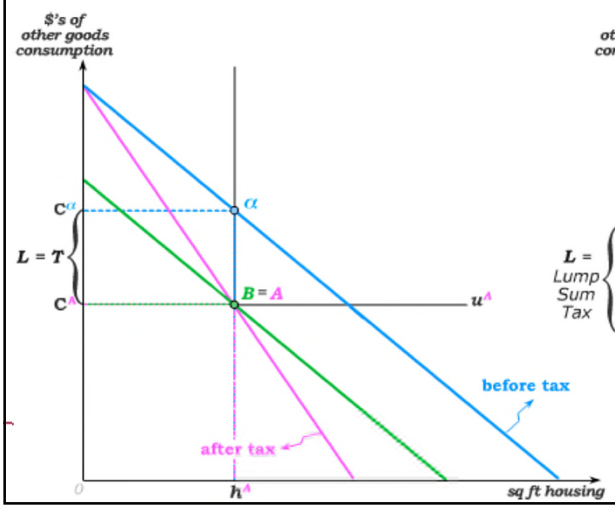


The vertical difference between the ***blue*** and ***green*** budget constraints is the most tax that could have been raised without making the consumer worse off …

… leaving us with the ***DWL***.

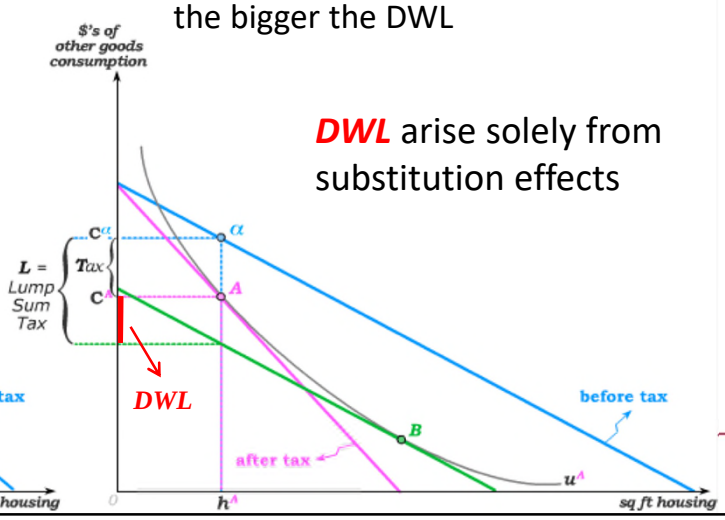
The effect of a tax III

Perfect complements have no DWL

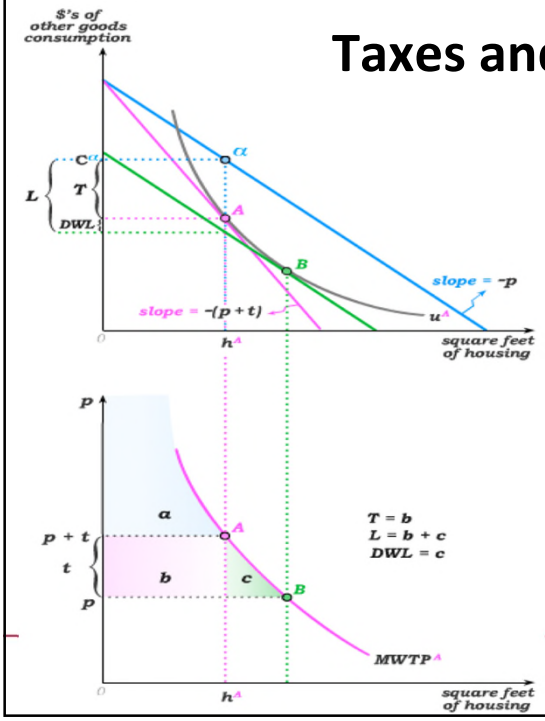


The greater the substitutability between taxed and non-taxed goods, the bigger the DWL

DWL arise solely from substitution effects



Taxes and Consumer Surplus



- When deriving MWTP, we do so from one indifference curve
- The tax moves the consumer down U_A from A to B, giving 2 points on $MWTP_A$

At A: Consumer surplus = ***a***

At B: Consumer surplus = ***a + b + c***

- As the consumer is indifferent between A and B, it must be that ***L = b + c***
- And $T = t \times h^A = b$, which implies
- *DWL* = ***L*** – ***T*** = ***c***.

Endowments of goods I

- An endowment: A bundle of goods owned by a consumer and tradable for other goods:(e_1, e_2)
- Say you have e_1 of good 1, but would choose $x_1 > e_1$
  - This means you are a net demander/buyer of good 1
  - If you would choose $x_1 < e_1$, you are a net supplier/seller of good 1
- Income is determined by your endowments and prices
- Consumer's choice set depends on endowments and prices

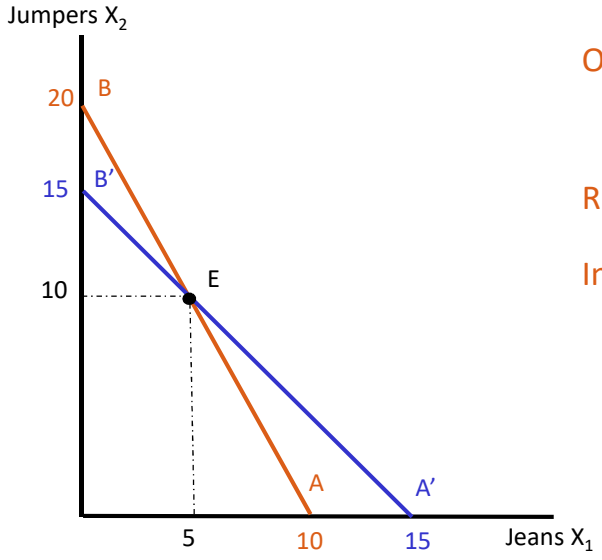
$$C(P_1, P_2, e_1, e_2) = \{(x_1, x_2) | P_1 x_1 + P_2 x_2 \leq P_1 e_1 + P_2 e_2\}$$

- The budget constraint will always pass through (e_1, e_2)

Endowments of goods II

- Say you bought 5 pairs of jeans (x_1) at £20 each and 10 jumpers (x_2) at £10 each from John Lewis for your partner
  - But they say that you bought the wrong ones and wrong quantities!
- You return to John Lewis, but don't have the receipt
  - You get a Gift Certificate for your goods at the current prices
  - $M = 5P_1 + 10P_2$
- If prices are unchanged, budget constraint is unchanged
- If prices have changed, budget constraint must still pass through (e_1, e_2) but will now have a different slope and intercepts

Endowments of goods III



Original budget constraint (AB):

$$P_1x_1 + P_2x_2 = 5P_1 + 10P_2$$

Rearranging gives: $x_2 = 5\frac{P_1}{P_2} + 10 - \frac{P_1}{P_2}x_1$

Intercept = 20 and Slope = -2

If jeans are on sale at 50% off, then Jeans in E are worth £10 not £20

Budget constraint pivots around E (A'B')

Intercept = 15 and slope = -1

Endogenous versus exogenous income

- Utility maximisation with exogenous income implies:
  - $x_1^*(p_1, p_2, M) = \frac{M}{2P_1}$ x_1 is independent of P_2
  - When P_1 changed, we held money income constant
- Utility maximisation with endogenous income implies:
  - $x_1^*(p_1, p_2, e_1, e_2) = \frac{\alpha M}{2P_1}$ where $M = p_1 e_1 + p_2 e_2$
  - x_1 now depends on P_1 and P_2
  - Now when P_1 changes, your money income changes too
- What happens to demand for x_1 when its price changes?

Slutsky: the endowment income effect I

$$\frac{\Delta x_1}{\Delta P_1} = \frac{x_1^s}{\Delta P_1} - \frac{\Delta x_1^m}{\Delta M} x_1$$

- Substitution effect remains the same
- But there are now two income effects to consider
  - Previously: say P_1 falls. Real income rises and so x_1 is affected (ordinary income effect: money income remains fixed)
  - But now when P_1 falls, your endowment and thus your money income is affected (endowment income effect)

$$\frac{\Delta x_1}{\Delta P_1} = \frac{x_1^s}{\Delta P_1} - \frac{\Delta x_1^m}{\Delta M} x_1 + \text{endowment income effect}$$

Slutsky: the endowment income effect II

- When P_1 changes, the price of the endowment changes and so money income changes and this changes demand

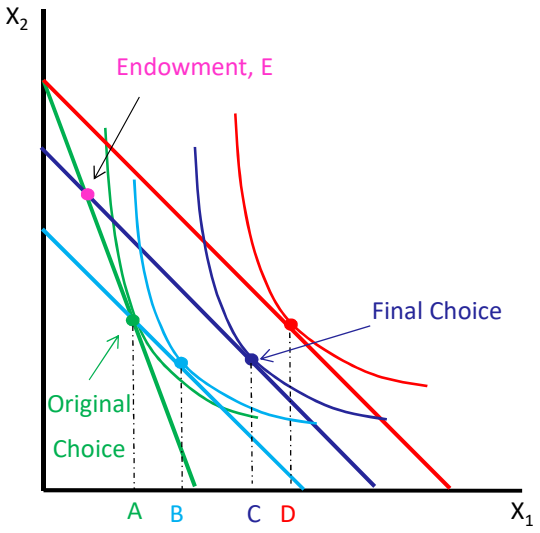
Endowment income effect = (i) Change in income when price changes x (i) Change in demand when income changes

- Consider term (i): We know: $P_1 e_1 + P_2 e_2 = M$
  - Change in income when price changes = $\frac{\Delta M}{\Delta P_1} = e_1$
- We already have an expression for term (ii): $\frac{\Delta x_1^m}{\Delta M}$

Slutsky: the endowment income effect III

- Endowment income effect: $\frac{\Delta x_1^m}{\Delta M}\frac{\Delta M}{\Delta P_1} = \frac{\Delta x_1^m}{\Delta M} e_1$
- Revised Slutsky equation: $\frac{\Delta x_1}{\Delta P_1} = \frac{x_1^s}{\Delta P_1} + (e_1 - x_1)\frac{\Delta x_1^m}{\Delta M}$
- Substitution effect is always negative: $P_1 \uparrow \rightarrow \downarrow x_1$
- With a normal good: ordinary income effect > 0
- Size of total effect depends on sign of $(e_1 - x_1)$:
  - Net demander: Price of normal good rises, demand must fall
  - Net supplier: It depends on the magnitude of the positive combined income effect, versus negative substitution effect

Endowments of goods IV



- Original budget constraint (A)
- Price of x_1 falls
- Substitution effect: A-B
- Ordinary income effect (holding money income fixed): B-D
- Endowment income effect (changes value of endowment and hence income): D-C
  - Must go through Endowment, E
- Final budget constraint (C)

Intertemporal Choice I

- Should you go to university?
  - No: You'll have a low income as a student and will incur huge debts
  - Yes: You'll have a higher income as a graduate and pensioner
- We can apply the consumer choice model to consider:
  - How much debt should you accumulate as a student?
  - How will the amount of debt depend on the interest rate?
  - How much should you save for retirement?
- This model is crucial to understanding saving decisions
- We treat two time periods (t = 1, 2) exactly like two goods

Intertemporal Choice II

- I can earn £10,000 (m_1) this summer and then travel next summer, earning £0 (m_2). Assume r = 10%
  - If c_1 = 0, I could consume $[m_1(1 + 0.1) + m_2]$ next summer.
  - For every £1 consumed today, next year's consumption falls by (1+r)
  - The most I have for consumption next summer is what I would have had if c_1 = 0 minus (1+r) times my actual consumption this summer

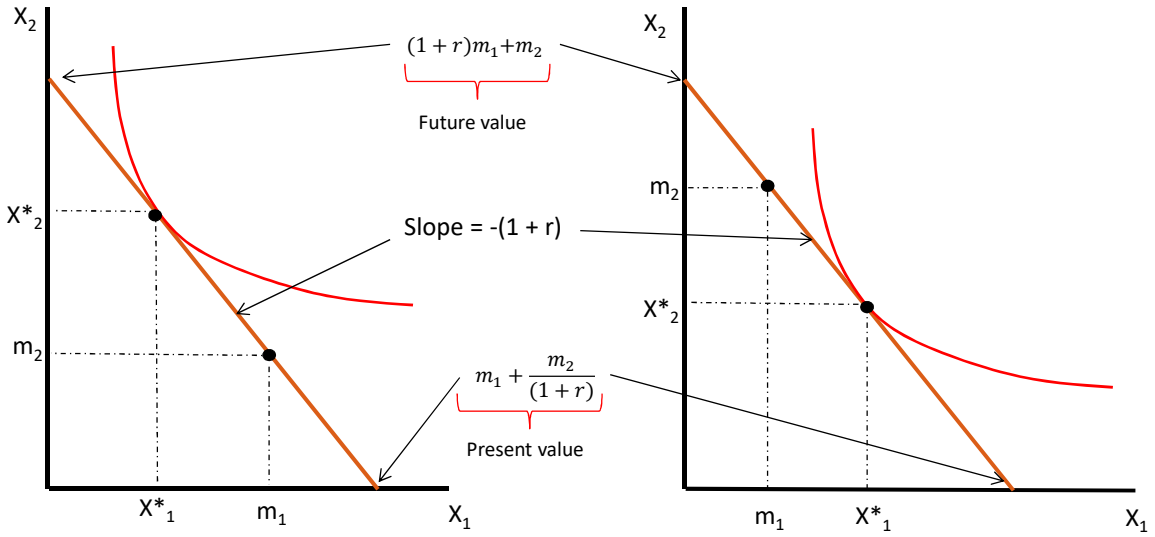
$$c_2 \leq m_1(1 + r) + m_2 - (1 + r)c_1$$

Rearranging gives:

$$(1 + r)c_1 + c_2 \leq m_1(1 + r) + m_2 \quad \text{Future value}$$

$$c_1 + c_2(1 + r)^{-1} \leq m_1 + m_2(1 + r)^{-1} \quad \text{Present value}$$

Lenders and Borrowers



Intertemporal Choice III

- Opportunity cost of consuming £1 today is (1+r) tomorrow
- In the N period model, opportunity cost is $(1 + r)^n$
- £10,000 invested today at interest rate r yields:
  - After 1 year: £10,000 (1 + r)
  - After 2 years: £10,000(1 + r)(1 + r) = 10,000$(1 + r)^2$
  - After n years: £10,000$(1 + r)^n$
- Budget Constraint becomes:

$$(1 + r)^n c_1 + c_n \leq m_1(1 + r)^n + m_n$$

  - This considers income and consumption in period 1 and n
  - What about the years in between?

Intertemporal Choice IV

- Assume I have an idea about my earning in all periods 1 – n
  - This gives n different endowments across n years $(m_1, m_2, \dots, m_n)$
  - Assume constant r across all years
- If I consume nothing until the last year, I have m_n plus:
  - Penultimate year's endowment and 1 year's interest: $m_n + (1 + r)m_{n-1}$
  - Plus second to last year's endowment and 2 year's interest, plus…
- So the maximum I could consume in last period is:

$$C_n = m_n + (1 + r)m_{n-1} + (1 + r)^2 m_{n-2} + \cdots + (1 + r)^{n-1} m_{n-(n-1)}$$

Intertemporal Choice V

- The actual amount I can consume depends on how much I consumed in the previous periods

$$C_n =$$

$$m_n + (1+r)m_{n-1} + (1+r)^2 m_{n-2} + \cdots + (1+r)^{n-1} m_1$$

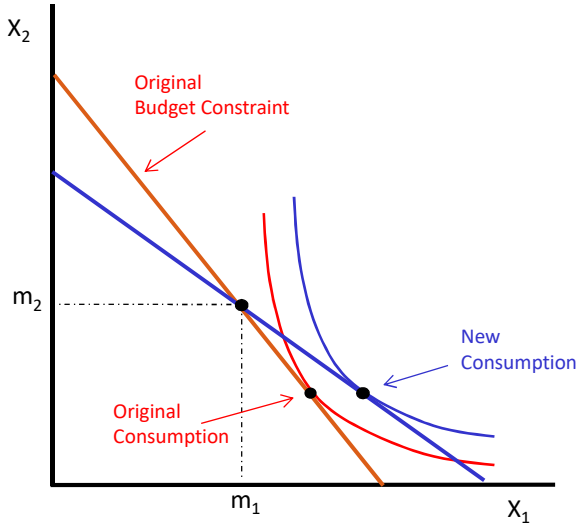
$$-(1+r)c_{n-1} - (1+r)^2 c_{n-2} - \cdots - (1+r)^{n-1} c_1$$

Or

$$C_n + (1+r)c_{n-1} + (1+r)^2 c_{n-2} + \cdots + (1+r)^{n-1} c_1$$

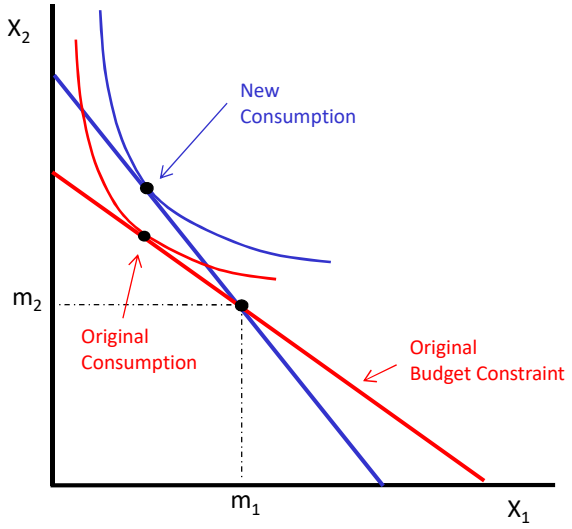
$$= m_n + (1+r)m_{n-1} + (1+r)^2 m_{n-2} + \cdots + (1+r)^{n-1} m_1$$

Comparative Statics: Borrowers



- What happens to the borrowing decision as 'r' changes?
- A decrease in 'r' pivots the budget constraint
- A borrower remains a borrower
- By Revealed preference, a borrower will be unambiguously better off
- If 'r' increases, a borrower may remain a borrower or may switch to become a lender

Comparative Statics: Lenders



- What happens to the borrowing decision as 'r' changes?
- An increase in 'r' pivots the budget constraint
- A lender remains a lender
- By Revealed preference, a lender will be unambiguously better off
- If 'r' decreases, a lender may remain a lender or may switch to become a borrower

Applications

- We can apply the consumer choice model to many areas and it can give important insights to policy-makers
  - Should taxes be increased on certain goods?
  - If interest rates change, how will this affect the behaviour of savers and borrowers?
  - If income tax rises, what happens to the supply of labour?
  - If in-work or out-of-work benefits change, how will this affect people's incentive to work?
- You will look at a further application in seminars