Chapter 9: Hypothesis Tests

9.1 Developing null & alternative hypotheses

Null hypothesis: tentative assumption about a population parameter, H_0.

Alternative hypothesis: opposite of what is stated in the null hypothesis, H_a or H_1.

NOTE:
Using hypothesis testing procedures, when H_0 is rejected, we have statistical evidence to support H_a.
When H_0 is *not* rejected, we have *no* statistical evidence to support either H_a, nor H_0.

TRICKS:
In the case of **research hypotheses,** it is easier to first identify H_a, and then develop H_0.
In the case of **assumptions to be challenged**, it is easier to first develop H_0, then specify H_a.

SUMMARY:

One-tail tests		**Two-tailed tests**
Lower Tail	Upper Tail	
H_0: $\mu \geq \mu_0$	H_0: $\mu \leq \mu_0$	H_0: $\mu = \mu_0$
H_a: $\mu < \mu_0$	H_a: $\mu > \mu_0$	H_a: $\mu \neq \mu_0$

where:
μ: population mean
μ_0: hypothesized value
either ≥, ≤ or = always appears in H_0

9.2 Type I & type II errors (see also text, 9.6 Hypothesis testing & decision making, & lecture)

We control the probability of making a Type I error, not a Type II, so to avoid the latter, we say:		Population Condition	
		H_0 true	**H_0 is not true**
Conclusion	**Do not reject H_0**	Correct conclusion	~~Type II error β~~
	Reject H_0	Type I error α	Correct conclusion

Level of significance, α: probability of making a Type I error.

9.3 Population mean: σ known

NOTE: The methods presented in this section are exact if the sample is selected from a population that is normally distributed. They are still applicable if the sample size is large enough.

Test statistic: $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$, where μ_0 is the hypothesized value of the population mean.

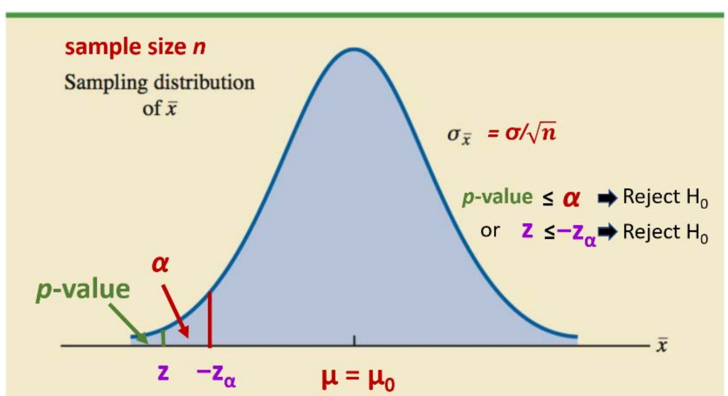
One-tailed test, Lower Tail Test:

***p*-value**: $P(\bar{x} \leq z)$ (see Ch.6.)

Critical value: $-z_\alpha$ such that $P(\bar{x} \leq -z_\alpha) = \alpha$

Rejection rule, *p*-value approach:
Reject H_0 if *p*-value ≤ α

Rejection rule, critical value approach:
Reject H_0 if $z \leq -z_\alpha$



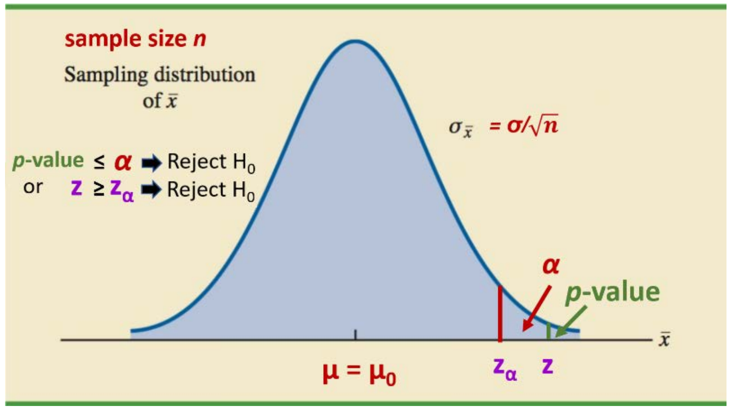
One-tailed test, Upper Tail Test:

***p*-value**: $P(\bar{x} \geq z)$ (see Ch.6.)

Critical value: z_α such that $P(\bar{x} \geq z_\alpha) = \alpha$

Rejection rule, *p*-value approach:
Reject H_0 if *p*-value $\leq \alpha$

Rejection rule, critical value approach:
Reject H_0 if $z \geq z_\alpha$



Two-tailed test:

***p*-value**: $2P(\bar{x} \geq |z|)$ (see Ch.6.)

Critical value: $z_{\alpha/2}$ such that
$P(\bar{x} \leq -z_{\alpha/2}) = \alpha/2$ and $P(\bar{x} \geq z_{\alpha/2}) = \alpha/2$

Rejection rule, *p*-value approach:
Reject H_0 if *p*-value $\leq \alpha$

Rejection rule, critical value approach:
Reject H_0 if $z \leq -z_{\alpha/2}$ or $z \geq z_{\alpha/2}$

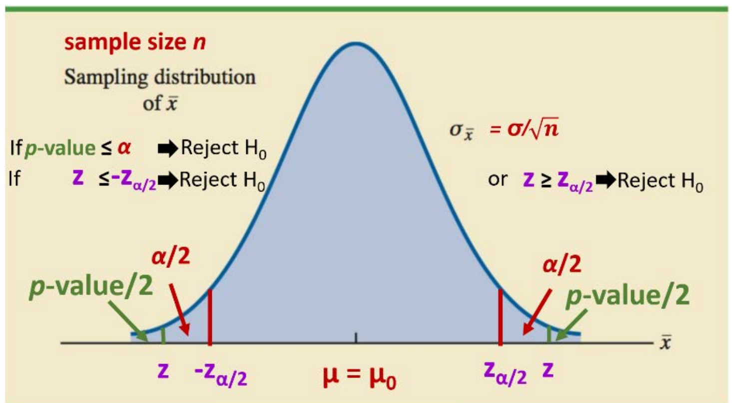


TABLE 9.2 SUMMARY OF HYPOTHESIS TESTS ABOUT A POPULATION MEAN: σ KNOWN CASE

	Lower Tail Test	Upper Tail Test	Two-Tailed Test
Hypotheses	$H_0: \mu \geq \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$
Test Statistic	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$
Rejection Rule: *p*-Value Approach	Reject H_0 if *p*-value $\leq \alpha$	Reject H_0 if *p*-value $\leq \alpha$	Reject H_0 if *p*-value $\leq \alpha$
Rejection Rule: Critical Value Approach	Reject H_0 if $z \leq -z_\alpha$	Reject H_0 if $z \geq z_\alpha$	Reject H_0 if $z \leq -z_{\alpha/2}$ or if $z \geq z_{\alpha/2}$

Relationship between interval estimation & hypothesis testing: *see text & lecture video.*

9.4 Population mean: σ *un*known

NOTE: The methods presented in this section are exact if the sample is selected from a population that is normally distributed. They are still applicable if the sample size is large enough.

TABLE 9.3 SUMMARY OF HYPOTHESIS TESTS ABOUT A POPULATION MEAN: σ UNKNOWN CASE

	Lower Tail Test	Upper Tail Test	Two-Tailed Test
Hypotheses	$H_0: \mu \geq \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$
Test Statistic	$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$
Rejection Rule: *p*-Value Approach	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$
Rejection Rule: Critical Value Approach	Reject H_0 if $t \leq -t_\alpha$	Reject H_0 if $t \geq t_\alpha$	Reject H_0 if $t \leq -t_{\alpha/2}$ or if $t \geq t_{\alpha/2}$

9.5 Population proportion

NOTE: We assume that $np \geq 5$ and $n(1 - p) \geq 5$, so we can use the normal probability distribution to approximate the sampling distribution of $\bar{p}$ (see Chapter 7).

TABLE 9.4 SUMMARY OF HYPOTHESIS TESTS ABOUT A POPULATION PROPORTION

	Lower Tail Test	Upper Tail Test	Two-Tailed Test
Hypotheses	$H_0: p \geq p_0$ $H_a: p < p_0$	$H_0: p \leq p_0$ $H_a: p > p_0$	$H_0: p = p_0$ $H_a: p \neq p_0$
Test Statistic	$z = \dfrac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1 - p_0)}{n}}}$	$z = \dfrac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1 - p_0)}{n}}}$	$z = \dfrac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1 - p_0)}{n}}}$
Rejection Rule: *p*-Value Approach	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$
Rejection Rule: Critical Value Approach	Reject H_0 if $z \leq -z_\alpha$	Reject H_0 if $z \geq z_\alpha$	Reject H_0 if $z \leq -z_{\alpha/2}$ or if $z \geq z_{\alpha/2}$

9.7 Calculating the probability of type II errors (see text and presentation)

Type II error, β: probability of accepting H_0 when H_0 is *not* true—when population mean is μ_a.

Power of a hypothesis test, 1 – β: probability of (correctly) rejecting H_0 when H_0 is not true.

9.8 Determining n to get specific α & β: **For a one-tailed test**: $n = (z_\alpha + z_\beta)^2\sigma^2/(\mu_0 - \mu_b)^2$ (see text for proof). **For a two-tailed**, use $z_{\alpha/2}$.