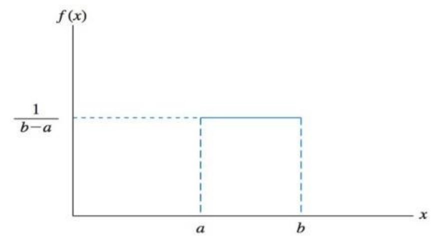
Chapter 6: Continuous Probability Distributions

For a discrete random variable, the probability function f(x) provides the probability that the random variable assumes a particular value. For a continuous random variable, the area under the graph of the probability density function f(x), corresponding to a given interval, provides the probability that the random variable assumes a value in that interval. This means (1) for a continuous random variable, the probability that the random variable assumes a particular value is zero and (2) the total area under the graph of f(x) is equal to 1 (since f(x) spans over all possible values of x).

6.1 Uniform Probability Distribution

Whenever the probability is proportional to the length of the interval, the random variable is uniformly distributed.



Uniform probability density function:

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$$

Expected value and variance for the uniform probability distribution:
E(x) = μ = $(a+b)/2$
Var(x) = $\sigma^2 = (b-a)^2/12$

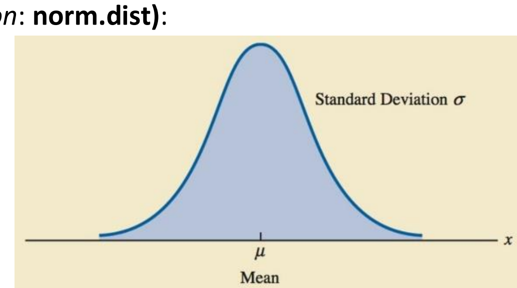
Proofs: see p.185, Example 3a, Ch.5.3 in Ross, 9th.

6.2 Normal Probability Distribution

Normal probability density function (*Microsoft Excel function*: **norm.dist)**:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where
μ = mean
σ = standard deviation
π = 3.14159
e = 2.71828



A random variable *z* is said to have a *standard* normal distribution when μ = 0 and σ = 1. Any normal random variable x can be transformed into a *standard* normal random variable: z = (x − μ)/σ. Note that z is the number of standard deviations σ that the normal random variable x is from its mean μ.

Proofs: see p.189, Example 4a, Ch.5.4 in Ross, 9th.

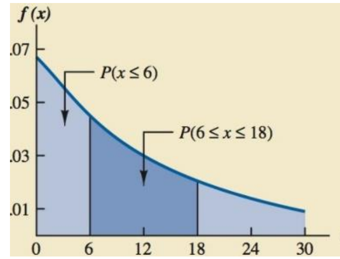
6.3 Normal Approximation of Binomial Probabilities

When $np \ge 5$ and $n(1-p) \ge 5$, set $\mu = np$ and $\sigma = \sqrt{np(1-p)}$ (Ch. 5) in definition of normal curve. To computing probabilities, use **continuity correction factor** (see text).

6.4 Exponential Probability Distribution

Exponential probability density function (*Microsoft Excel function*: **expon.dist)**:

$f(x) = \frac{1}{\mu} e^{-x/\mu}$ for $x \geq 0$



where

μ = mean

NOTE: In Excel, parameter $\lambda = 1/\mu$.

Variance for the exponential probability distribution:

$Var(x) = \sigma^2 = \mu^2$

Proofs: see p.198, Example 5a, Ch.5.5 in Ross, 9th.