



The Normal Distribution

Stats Tutor London



Objectives

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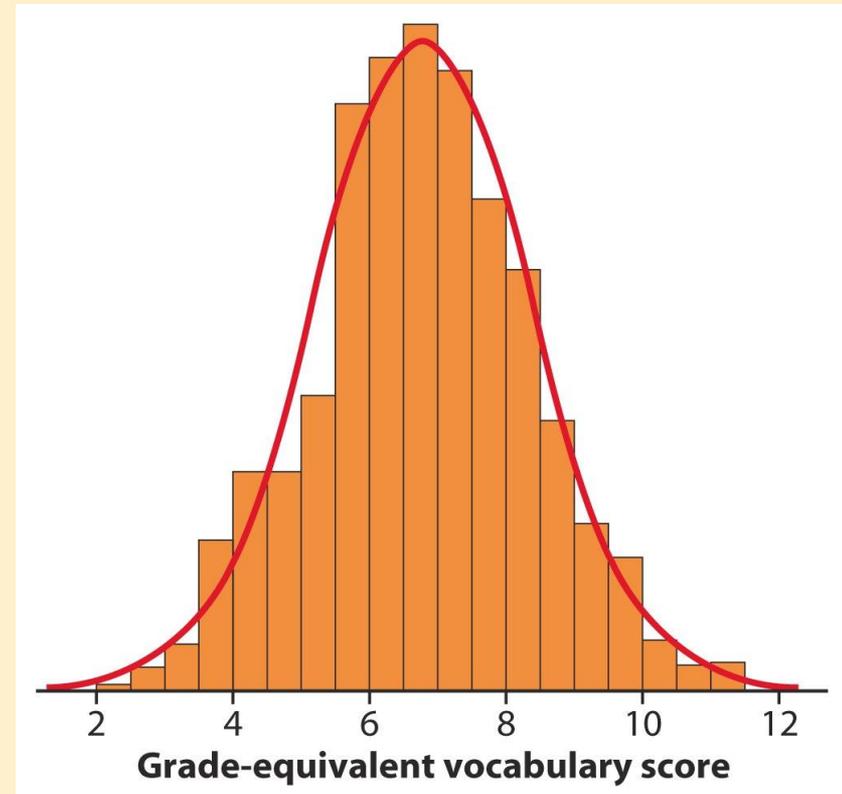
- Define and describe density curves
- Measure position using percentiles
- Measure position using z-scores
- Describe Normal distributions
- Describe and apply the 68-95-99.7 Rule
- Describe the standard Normal distribution
- Perform Normal calculations

Density Curves

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Sometimes the overall pattern of a large number of observations is so regular that we can describe it by a smooth curve.

Example: Here is a histogram of vocabulary scores of 947 seventh graders.



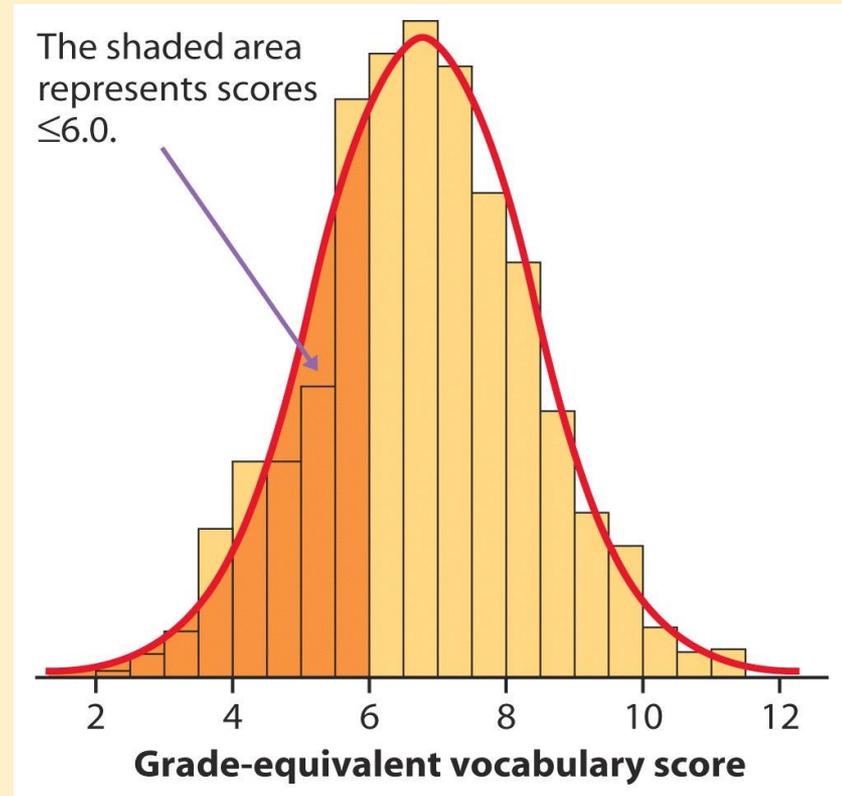
The smooth curve drawn over the histogram is a **mathematical model** for the distribution.

Density Curves

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The areas of the shaded bars in this histogram represent the proportion of scores in the observed data that are less than or equal to 6.0. This proportion is equal to 0.303.

Now the area under the smooth curve to the left of 6.0 is shaded. If the scale is adjusted so the total area under the curve is exactly 1, then this curve is called a **density curve**. The proportion of the area to the left of 6.0 is now equal to 0.293.



Density Curves

A **density curve** is a curve that:

- is always on or above the horizontal axis
- has an area of exactly 1 underneath it

A density curve describes the overall pattern of a distribution. The area under the curve and above any range of values on the horizontal axis is the proportion of all observations that fall in that range.

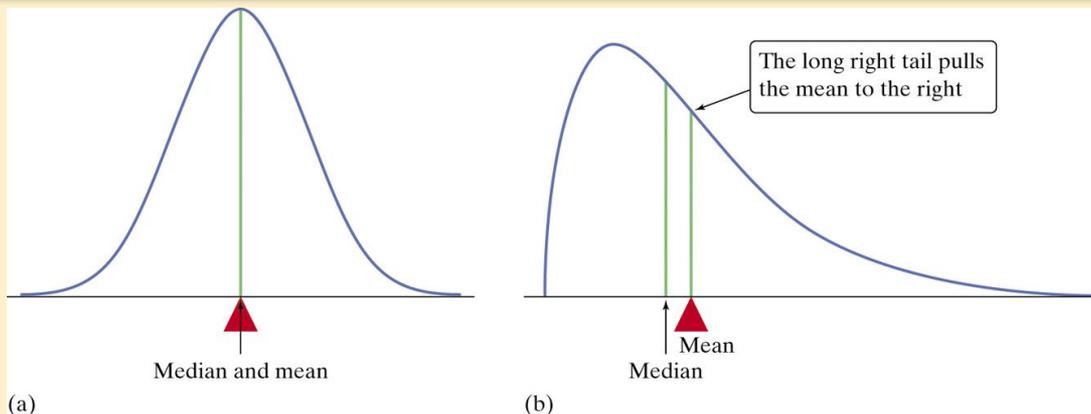
Density Curves

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- Our measures of center and spread apply to density curves as well as to actual sets of observations.

Distinguishing the Median and Mean of a Density Curve

- The **median** of a density curve is the equal-areas point, the point that divides the area under the curve in half.
- The **mean** of a density curve is the balance point, at which the curve would balance if made of solid material.
- The median and the mean are the same for a symmetric density curve. They both lie at the center of the curve. The mean of a skewed curve is pulled away from the median in the direction of the long tail.



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Density Curves

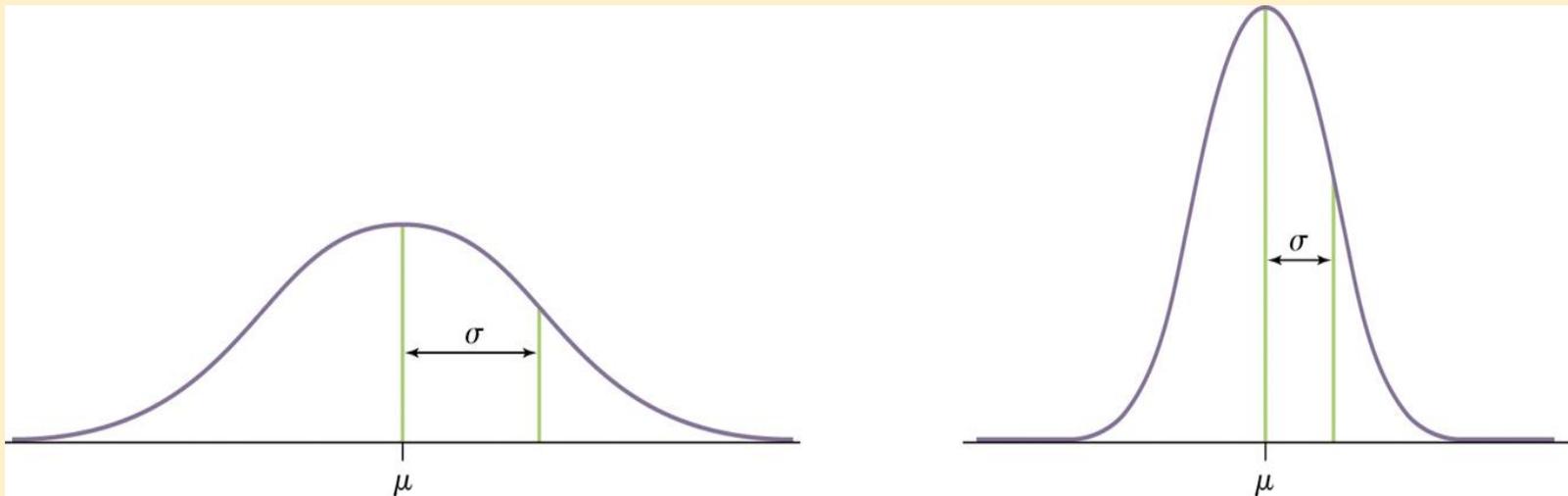
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- The mean and standard deviation computed from actual observations (data) are denoted by \bar{x} and s , respectively.
- The mean and standard deviation of the distribution represented by the density curve are denoted by μ (“myu”) and σ (“sigma”), respectively.

Normal Distributions

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- One particularly important class of density curves are the Normal curves, which describe Normal distributions.
- All Normal curves are symmetric, single-peaked, and bell-shaped
- A Specific Normal curve is described by giving its mean μ and standard deviation σ .



Normal Distributions

A **Normal distribution** is described by a Normal density curve. Any particular Normal distribution is completely specified by two numbers: its mean μ and standard deviation σ .

- The mean of a Normal distribution is the center of the symmetric **Normal curve**.
- The standard deviation is the distance from the center to the change-of-curvature points on either side.
- We abbreviate the Normal distribution with mean μ and standard deviation σ as $N(\mu, \sigma)$.

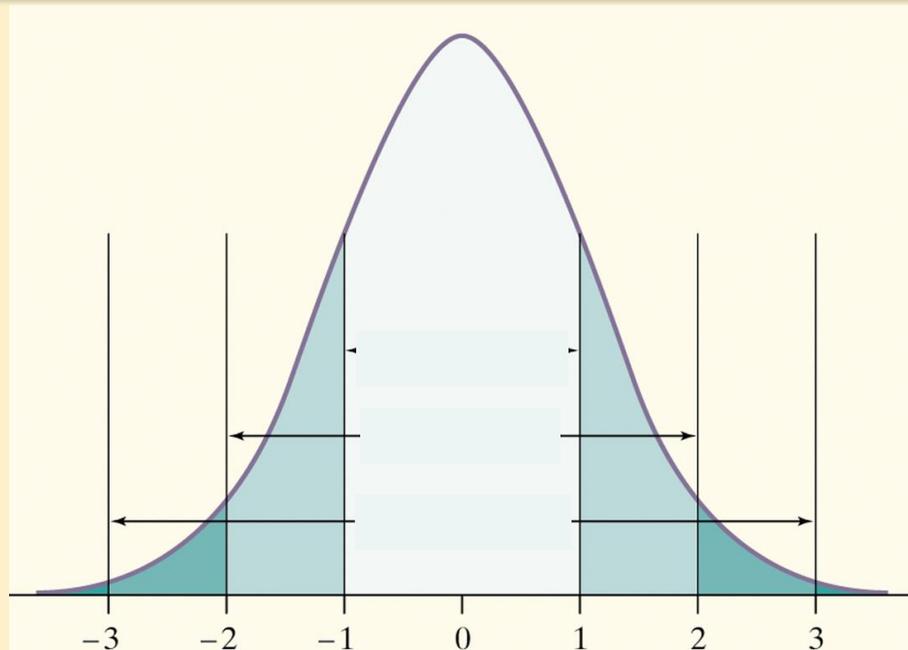
The 68-95-99.7 Rule

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The 68-95-99.7 Rule

In the Normal distribution with mean μ and standard deviation σ :

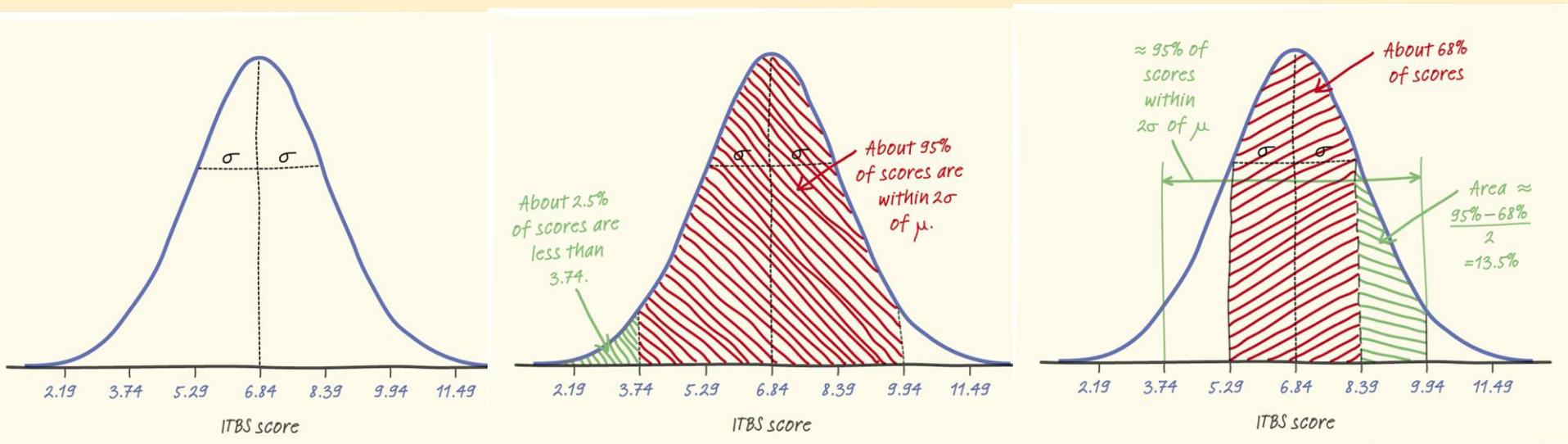
- Approximately **68%** of the observations fall within σ of μ .
- Approximately **95%** of the observations fall within 2σ of μ .
- Approximately **99.7%** of the observations fall within 3σ of μ .



Normal Distributions

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- The distribution of Iowa Test of Basic Skills (ITBS) vocabulary scores for 7th-grade students in Gary, Indiana, is close to Normal. Suppose the distribution is $N(6.84, 1.55)$.
- Sketch the Normal density curve for this distribution.
- What percent of ITBS vocabulary scores are less than 3.74?
- What percent of the scores are between 5.29 and 9.94?



The Standard Normal Distribution

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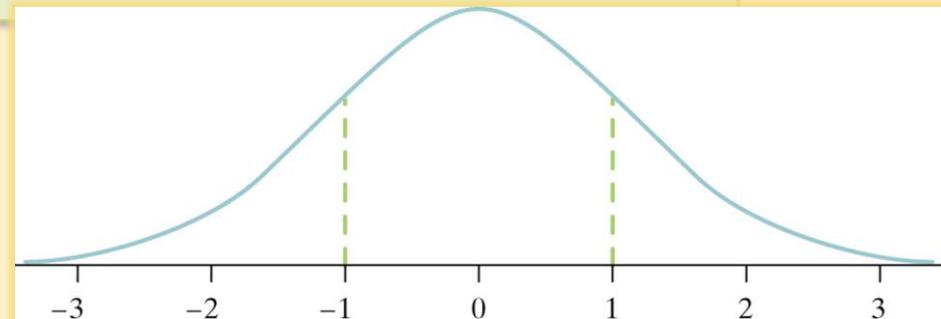
- All Normal distributions are the same if we measure in units of size σ from the mean μ as center.

The **standard Normal distribution** is the Normal distribution with mean 0 and standard deviation 1.

If a variable x has any Normal distribution $N(\mu, \sigma)$ with mean μ and standard deviation σ , then the standardized variable

$$z = \frac{x - \mu}{\sigma}$$

has the standard Normal distribution, $N(0,1)$.



The Standard Normal Table

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- Because all Normal distributions are the same when we standardize, we can find areas under any Normal curve from a single table.

The Standard Normal Table

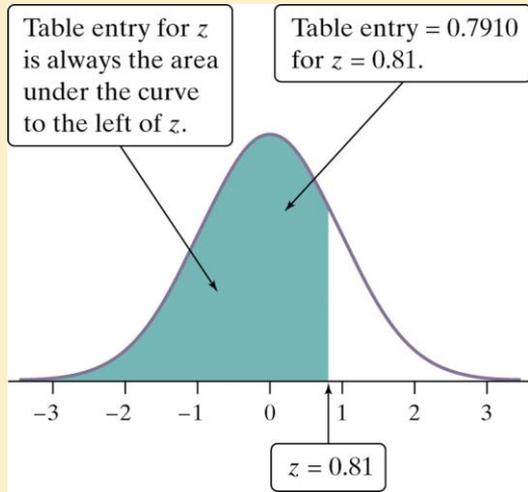
Table A is a table of areas under the standard Normal curve. The table entry for each value z is the area under the curve to the left of z .

Suppose we want to find the proportion of observations from the standard Normal distribution that are less than 0.81.

We can use Table A:

Z	.00	.01	.02
0.7	.7580	.7642	.7642
0.8	.7881	.7910	.7939
0.9	.8159	.8186	.8212

$$P(z < 0.81) = .7910$$

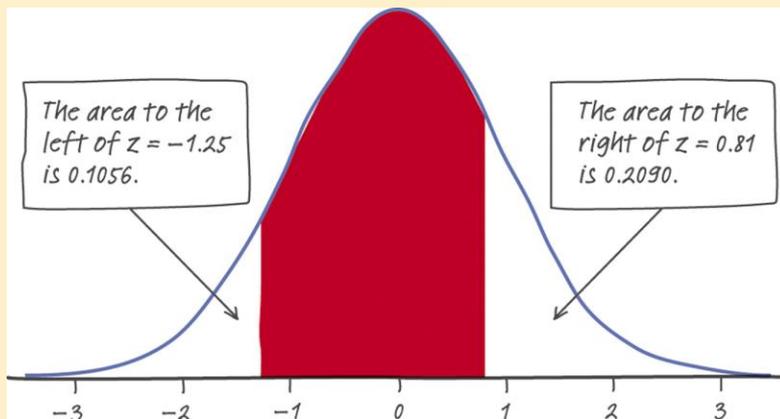


Normal Calculations

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Find the proportion of observations from the standard Normal distribution that are between -1.25 and 0.81.

Can you find the same proportion using a different approach?



$$1 - (0.1056 + 0.2090) = 1 - 0.3146 \\ = \mathbf{0.6854}$$

Normal Calculations

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How to Solve Problems Involving Normal Distributions

State: Express the problem in terms of the observed variable x .

Plan: Draw a picture of the distribution and shade the area of interest under the curve.

Do: Perform calculations.

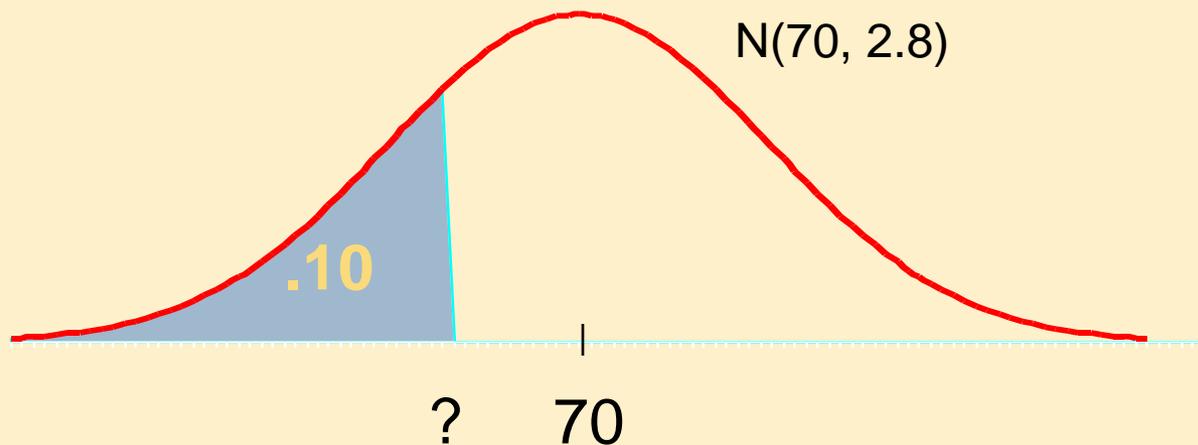
- **Standardize** x to restate the problem in terms of a standard Normal variable z .
- **Use the standard normal table** and the fact that the total area under the curve is 1 to find the required area under the standard Normal curve.

Conclude: Write your conclusion in the context of the problem.

Normal Calculations

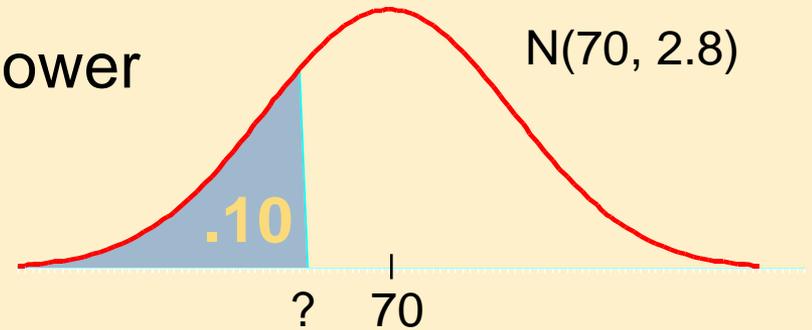
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- According to the Health and Nutrition Examination Study of 1976-1980, the heights (in inches) of adult men aged 18-24 are $N(70, 2.8)$.
- How tall can a man be in the lower 10% for men aged 18 to 24?



Normal Calculations

- How tall must a man be in the lower 10% for men aged 18 to 24?



Look up the closest probability (closest to 0.10) in the table.

Find the corresponding **standardized score**.

The value you seek is that many standard deviations from the mean.

z	.07	.08	.09
-1.3	.0853	.0808	.0823
-1.2	.1003	.0985	.0985
-1.1	.1210	.1190	.1170

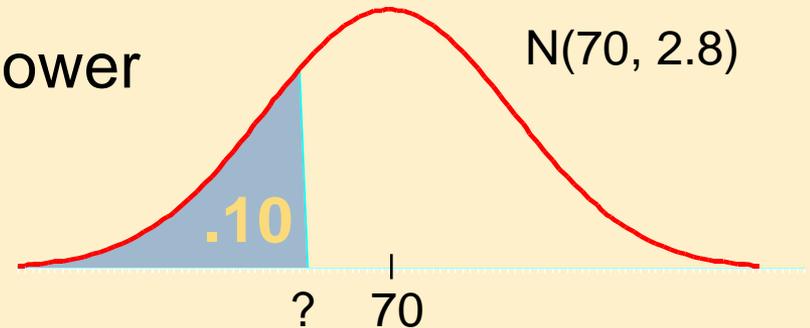
Z = -1.28

Normal Calculations

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- How tall must a man be in the lower 10% for men aged 18 to 24?

$$Z = -1.28$$



We need to “unstandardize” the z-score to find the observed value (x):

$$z = \frac{x - \mu}{\sigma} \quad \Longrightarrow \quad x = \mu + z\sigma$$

$$\begin{aligned} x &= 70 + z(2.8) \\ &= 70 + [(-1.28) \times (2.8)] \\ &= 70 + (-3.58) = \underline{\underline{66.42}} \end{aligned}$$

A man would have to be approximately 66.42 inches tall or less to place in the lower 10% of all men in the population.

Objectives Review

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