The University of Nottingham

School of Economics

Microeconomics Tutor Nottingham

Utility Functions

Lecture Overview

- Lecture 4: representation of consumers' preferences using indifference curve.
- Preferences can also be represented mathematically using utility functions.
- Looked at the necessary mathematical tools in the last lecture.
- Now apply these tools to the analysis of utility functions.

Utility Functions

- Recall: Our model of consumer behaviour implies that consumers can compare bundles of goods.
- Can summarise preferences by assigning a numerical value (‘utility’) to each bundle.
- A utility function U(x) assigns a utility level to every possible bundle of goods.
- Allows comparison of bundles
  - $U(x) > U(y)$ is equivalent to $x \succ y$.
  - $U(x) = U(y)$ is equivalent to $x \sim y$.

Utility Functions

- Example: Suppose Lisa's preferences over pizzas and burritos can be represented by the following utility function:

$$U(q_P, q_B) = {q_P}^{0.5} {q_B}^{0.5} = \sqrt{q_P q_B}$$

- If bundle x has 72 burritos and 8 pizzas and bundle y has 12 each, Lisa prefers x to y because U(x) = 24 > U(y) = 12.

Utility Functions

- Two types of utility functions: ordinal and cardinal.
- Ordinal utility functions:
  - Only describe rankings of bundles, not utility *levels*.
  - If $U(x) = 2 \times U(y)$, ***does not*** imply Lisa likes *x* twice as much as *y*.
  - Different utility functions can represent the same preferences (e.g., $V(q_P, q_B) = 2 \times \sqrt{q_P q_B}$)
  - Do not allow interpersonal comparisons of utility.

Utility Functions

- Cardinal utility functions assign exact utility levels to bundles.
  - If U(x) is cardinal, $U(x) = 2 \times U(y)$ ***does*** imply that Lisa likes x twice as much as y.
- We will mostly use ordinal utility functions:
  - People can usually only rank bundles, not assign exact utility levels to them.
  - Interpersonal comparisons of utility generally thought of as impossible.

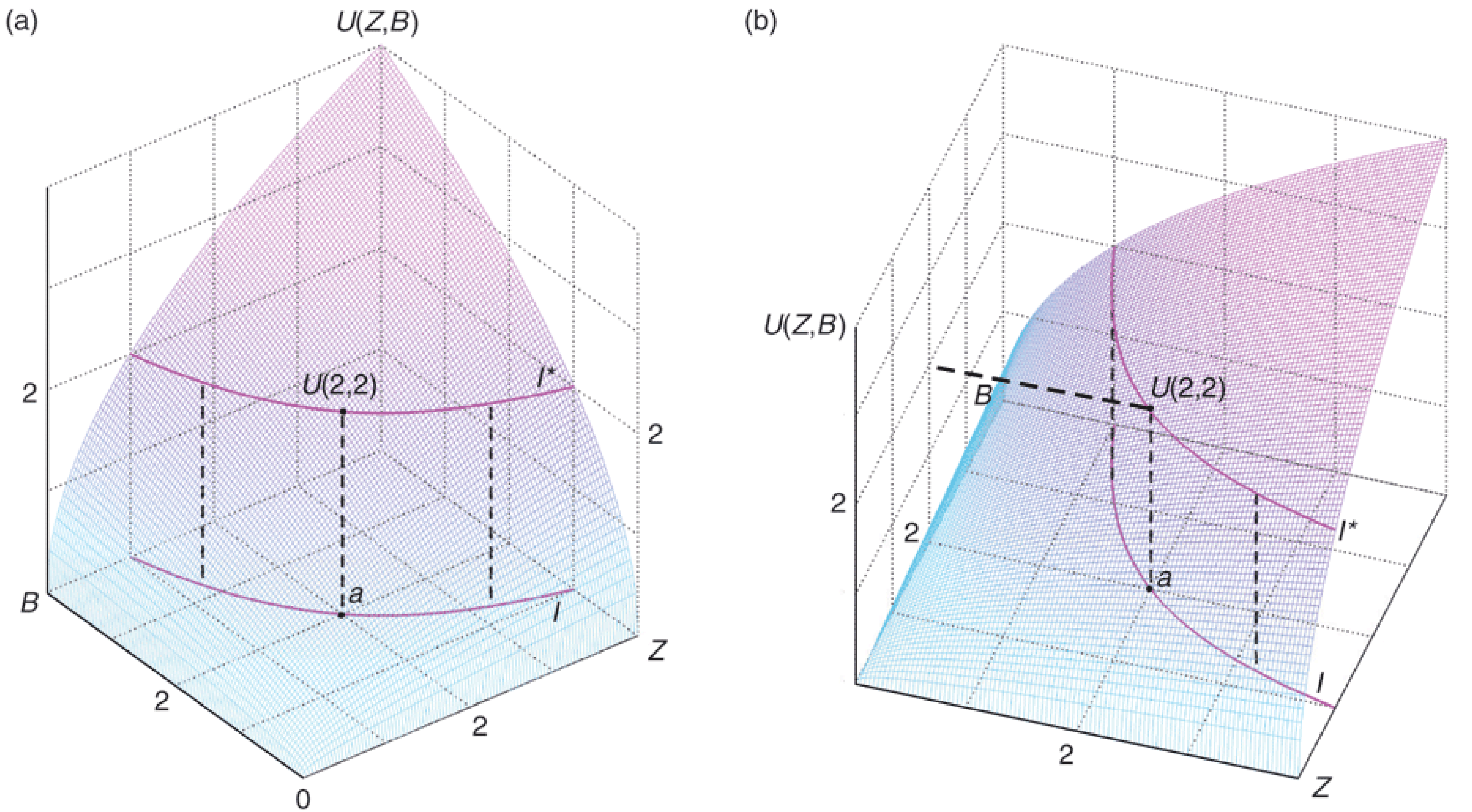
Utility Functions

- Utility functions can summarise the information in indifference maps.
- If a consumer's utility is $\mathrm{U}(q_1, q_2)$ then one of the corresponding indifference curves is:

$$\overline{U}=\mathrm{U}(q_1, q_2)$$

- Interpretation as contour lines of a three-dimensional plot of $\mathrm{U}(q_1, q_2)$. (Figure 1)

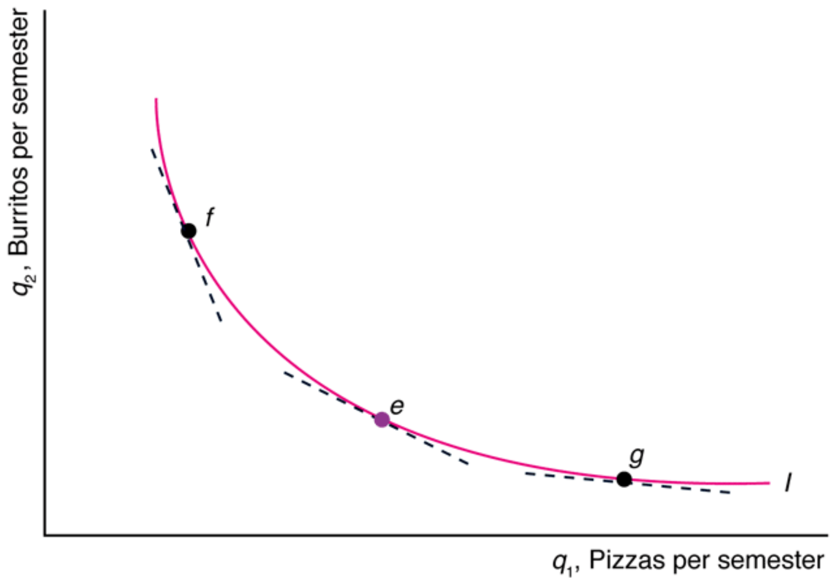
Figure 1: Utility Functions



Utility function: U(Z,B)

Utility Functions

- Utility functions allow a reinterpretation of the slope of indifference curves.
- Recall that the slope measures the marginal rate of substitution (MRS) between goods.



$$\text{MRS} = slope = \frac{dq_2}{dq_1}$$

Utility Functions

- Start by defining the ***marginal utility*** of a good.
- Change in total utility from consuming an extra unit of a good.

Slices of pizza	**Total utility**	**Marginal utility**
0	0	--
1	20	20
2	38	18
3	53	15
4	61	8
5	65	4
6	55	-10

Utility Functions

- Mathematically,

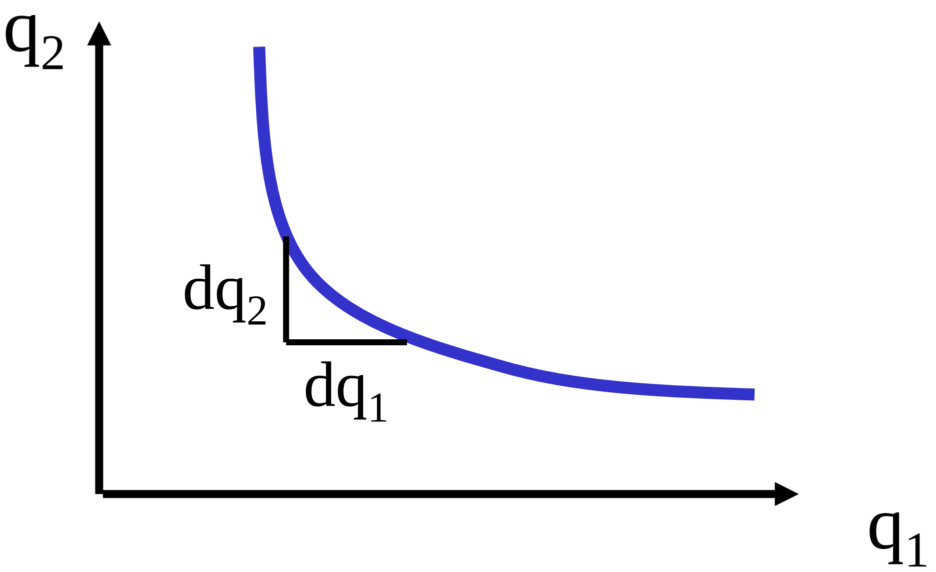
$$U_1 = \frac{\partial \mathrm{U}(q_1,q_2)}{\partial q_1} \qquad U_2 = \frac{\partial \mathrm{U}(q_1,q_2)}{\partial q_2}.$$

- Example: for $U(q_1, q_2) = {q_1}^{0.5}{q_2}^{0.5}$, we have

$$U_1 = 0.5{q_1}^{-0.5}{q_2}^{0.5}, \; U_2 = 0.5{q_1}^{0.5}{q_2}^{-0.5}.$$

Utility Functions

- Moving down the curve, we give up q_2 and gain q_1.



- Giving up a small amount of q_2 (dq_2) decreases utility by $\frac{\partial U(q_1,q_2)}{\partial q_2} dq_2$; gaining dq_1 increases it by $\frac{\partial U(q_1,q_2)}{\partial q_1} dq_1$.

Utility Functions

- Along the indifference curve utility is constant:

$$\frac{\partial \mathrm{U}(q_1, q_2)}{\partial q_1} dq_1 + \frac{\partial \mathrm{U}(q_1, q_2)}{\partial q_2} dq_2 = 0$$

- Solving for the MRS:

$$\mathrm{MRS} = \frac{dq_2}{dq_1} = -\frac{\partial \mathrm{U}(q_1, q_2)}{\partial q_1} \bigg/ \frac{\partial \mathrm{U}(q_1, q_2)}{\partial q_2} = -\frac{U_1}{U_2}$$

Utility Functions

- So the slope of an indifference curve is equal to the ratio of marginal utilities (will become important later).
- If indifference curves are convex, so that the MRS is diminishing:
  - $|U_1/U_2|$ becomes smaller as we move down the curve.
  - As we get more q_1 each additional unit is worth less to us (so U_1 goes down).
  - As we get less q_2 each additional unit is worth more to us (so U_2 goes up).

Utility Functions

Consider Lisa's utility function over pizzas and burritos, $U(q_P, q_B) = {q_P}^{0.5}{q_B}^{0.5}$. Which of the following statements is **true**?

A. Lisa prefers two pizzas and one burrito to one pizza and two burritos.

B. The marginal utility of pizza is $U_P = 0.5{q_P}^{0.5}{q_B}^{-0.5}$.

C. The marginal utility of burritos is $U_B = {q_P}^{0.5}{q_B}^{-0.5}$.

D. The marginal rate of substitution between pizzas and burritos is diminishing.

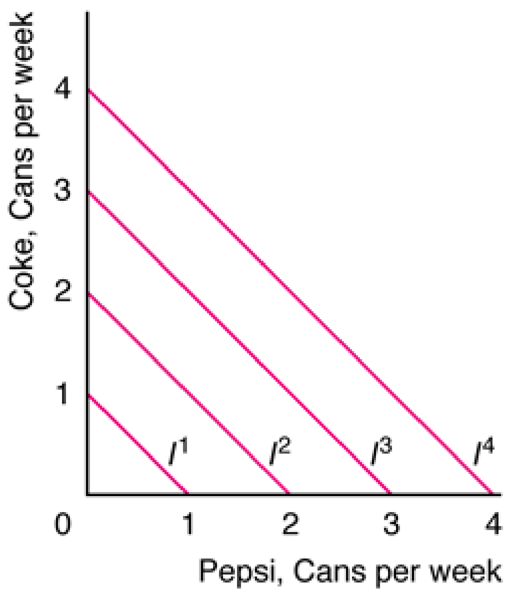
E. The marginal rate of substitution between pizzas and burritos is constant.

Utility Functions

Examples of often-used utility functions:

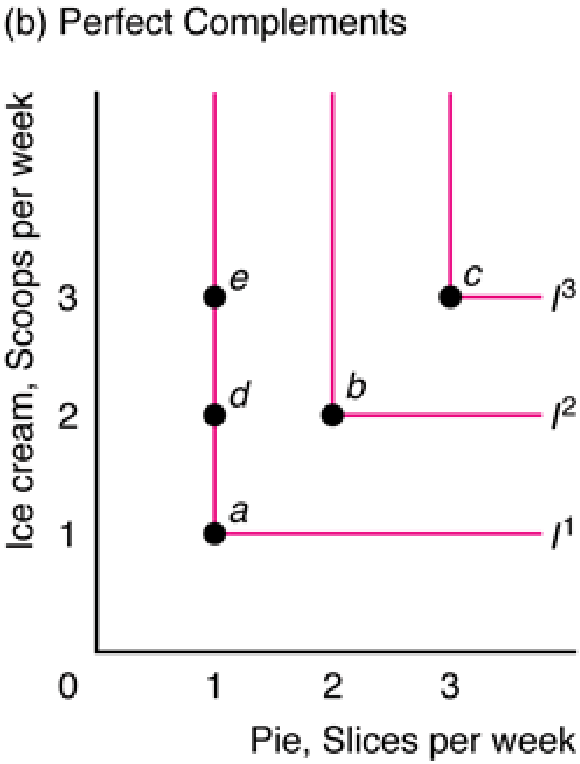
- Perfect substitutes
  - Goods that a consumer is indifferent about.
  - Examples: Coke vs Pepsi, mineral water, generic vs brand-name drugs …
  - $U(q_1, q_2) = iq_1 + jq_2$
  - $MRS = i/j$

(a) Perfect Substitutes



Utility Functions

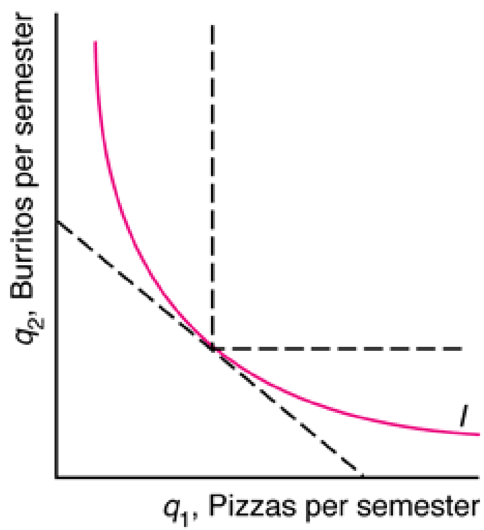
- Perfect complements
  - Goods that are consumed in fixed proportions.
  - Examples: iPods/headphones, apple pie/ice cream …
  - $U(q_1, q_2) = min(iq_1, jq_2\,)$
  - MRS is not defined ($U_1 = 0$, $U_2 = 0$, unwilling to substitute goods for each other).

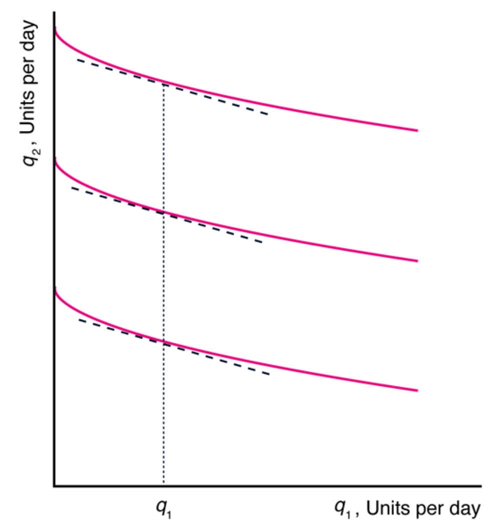


Utility Functions

- Imperfect substitutes
  - Are between the extreme examples of perfect substitutes and perfect complements.
  - Many types of utility functions which yield standard-shaped, convex indifference curves. Two examples:

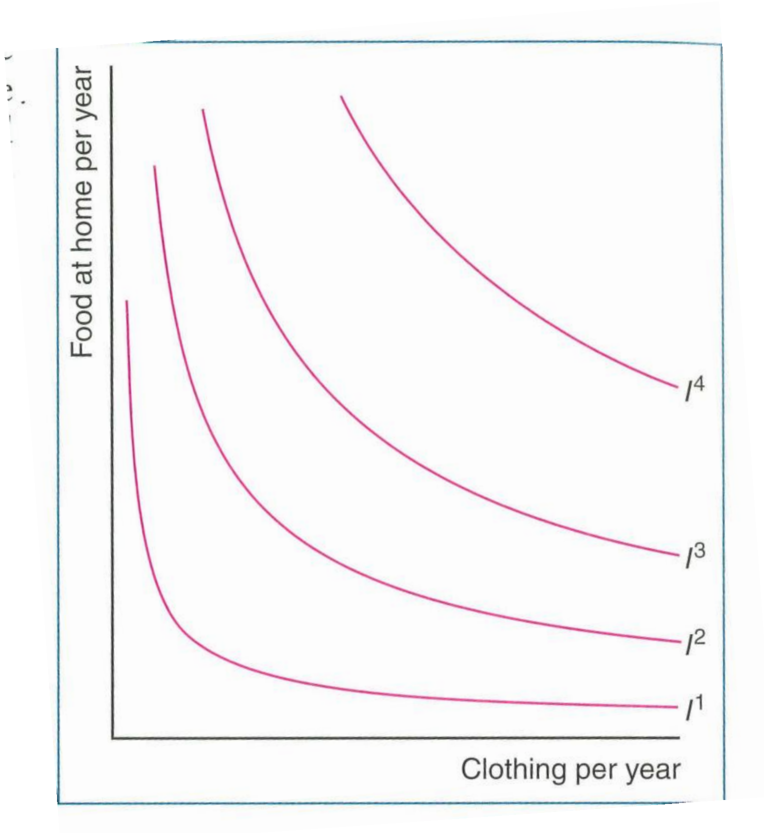
Cobb-Douglas: $U(q_1, q_2) = {q_1}^{a} {q_2}^{1-a}$ Quasilinear: $U(q_1, q_2) = u(q_1) + q_2$





Utility Functions

Consider the indifference curves estimated by Eastwood and Craven (1981) for the average U.S. consumers over food consumption and clothing. Which of the following statements ***does not*** follow from the diagram below?



A) We approach the case of perfect complementarity for low levels of consumption.
B) Consumers become more indifferent between the two goods as they consume more of them.
C) The fact that a minimum level of food and clothing is necessary to support life can help explain the shapes of the indifference curves.
D) Consumers care less about food and clothing as they get richer.
E) Food and clothing are imperfect substitutes in the diagram.

Summary & Learning Outcomes

- Utility functions allow a convenient mathematical representations of preferences.
- Utility functions and indifference curves can be used to model many types of goods (substitutes, complements …).

Summary & Learning Outcomes

- Understand the nature of cardinal and ordinal utility functions.
- Understand the link between utility functions and indifference curves.
- Know the main types of utility functions and the associated indifference curves and MRSs.

Reading

- Perloff: chapter 3.2.
- Morgan, Katz and Rosen: chapter 2.2 (pages 40-43) and Appendix 3.A.1