Interest Rate and Bond Valuation

Investment Tutor London

Key Concepts and Skills

- Know the important bond features and bond types
- Understand:
  - Bond values and why they fluctuate
  - Bond ratings and what they mean
  - The impact of inflation on interest rates
  - The term structure of interest rates and the determinants of bond yields

Bond Definitions

A bond: "A certificate of debt, generally *long term*, under the terms of which an issuer contracts, among other things, to pay the holder a *fixed principal amount* on a *stated future date* and, usually, a *series of interest payments* during its life"

Generally speaking, the bond has an intrinsic value: the stated cash flows; hence, bond can be traded in the secondary markets.

Key Features of a Bond

- Par value (sometimes called face value or principal amount)
- Coupon rate
- Maturity date
- Yield to maturity (YTM)

Key Features of a Bond

Par value & coupon rate

Par value:

- Face amount
- Re-paid at maturity

Coupon rate: the rate of interest that the borrower pays the bondholder. It is expressed as an annual percentage of the face value

- Multiply by par value to get coupon payment
- Annual vs. semi-annual

$$\text{Coupon amount} = \frac{\text{coupon rate}/\text{no. of coupons per year}}{100} \times \text{par value}$$

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Coupon Rate

Example

A bond has a face value of \$1,000, a coupon rate of 10% and 10 years to maturity. Assuming annual coupon payments, what is the amount of each coupon payment?

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Assuming semi-annual coupon payments, what is the amount of each coupon payment?

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$$\begin{aligned} C &= 10\% \times \$1,000 \\ &= \$100 \end{aligned}$$

Assuming semi-annual coupon payments, what is the amount of each coupon payment?

Answer:

$$\begin{aligned} C &= \frac{10\%}{2} \times \$1,000 \\ &= 5\% \times 1,000 \\ &= \$50 \end{aligned}$$

Key Features of a Bond
(cont'd)

Maturity: the final date by which bondholders receive all the principal due, as well as the interest.

Yield to maturity (YTM):

- The market required rate of return for bonds of similar risk and maturity
- The discount rate used to value a bond
- Return if the bond is held to maturity and all the coupon payments are reinvested at YTM
- Quoted as an APR

Bond Value

So, how do we value bonds?

$$\text{Bond value} = PV\,(\text{coupons}) + PV\,(\text{par})$$

$$\text{Bond value} = PV\,(\text{annuity}) + PV\,(\text{lump sum})$$

Remember:

As interest rates increase present values decrease, i.e., $r \uparrow \Rightarrow PV \downarrow$
As interest rates increase, bond prices decrease and vice versa

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The Bond-Pricing Equation

Where BV = bond value, C = coupon payment, F = face value (par value), YTM = yield to maturity and t = years to maturity:

$$BV = C\left[\frac{1-\frac{1}{(1+YTM)^t}}{YTM}\right]+\frac{F}{(1+YTM)^t}$$

It is worthwhile comparing the bond-pricing equation to the formula for an annuity:

$$PV = PMT\left[\frac{1-\frac{1}{(1+r)^t}}{r}\right]$$

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Valuing a Discount Bond with Annual Coupons

Example

Price the following bond: coupon rate $= 10\%$; annual coupons; par $= \$1,000$; maturity $= 5$ years; $YTM = 11\%$.

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Valuing a Discount Bond with Annual Coupons

Example

Price the following bond: coupon rate $= 10\%$; annual coupons; par $= \$1,000$; maturity $= 5$ years; $YTM = 11\%$.

Answer:

$$
\begin{aligned}
BV &= (0.10 \times 1,000)\left[\frac{1-\frac{1}{(1.11)^5}}{0.11}\right] + \frac{1,000}{(1.11)^5} \\
&= 100 \times 3.6959 + 593.4513 \\
&= \$963.04
\end{aligned}
$$

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Valuing a Premium Bond with Annual Coupons

Example

If $YTM = 8\%$, price the bond in the previous example.

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Valuing a Premium Bond with Annual Coupons

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If $YTM = 8\%$, price the bond in the previous example.

Answer:

$$\begin{aligned} BV &= (0.10 \times 1,000)\left[\frac{1 - \frac{1}{(1.08)^5}}{0.08}\right] + \frac{1,000}{(1.08)^5} \\ &= 100 \times 3.9927 + 680.58 \\ &= \$1,079.85 \end{aligned}$$

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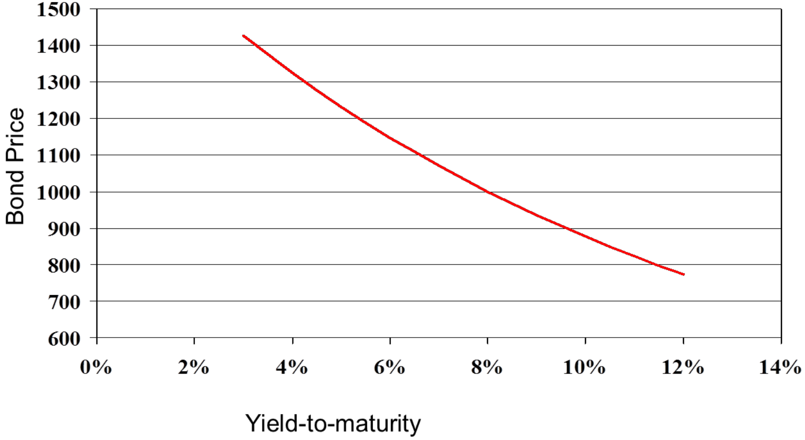
Relationship Between Price and Yield-to-Maturity



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Relationship Between Price and Yield-to-Maturity
(cont'd)

YTM > Coupon rate	Price < Par	"Discount bond"
YTM < Coupon rate	Price > Par	"Premium bond"
YTM = Coupon rate	Price = Par	n/a

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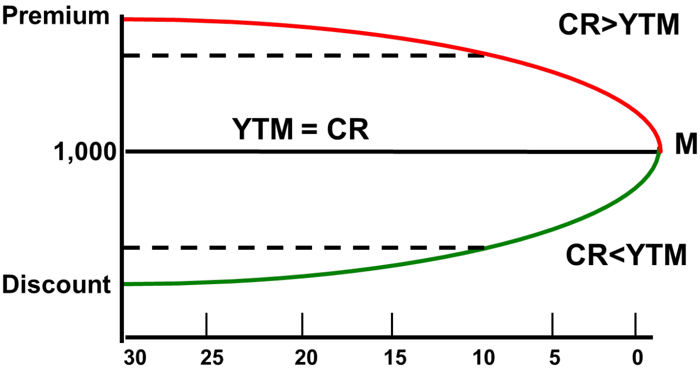
Bond Value & Time Remaining to Maturity



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The Bond-Pricing Equation

Adjusted for Semi-Annual Coupons

$$BV = \frac{C}{2}\left[\frac{1 - \frac{1}{(1+YTM/2)^{2t}}}{YTM/2}\right] + \frac{F}{(1+YTM/2)^{2t}}$$

where

BV = bond value

C = coupon payment, $C/2$ = semi-annual coupon

F = face value (par value)

YTM = yield to maturity, $YTM/2$ = semi-annual YTM

t = years to maturity, $2t$ = number of 6-month periods to maturity

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Valuing Semi-Annual Bonds

Example

Price the following bond: coupon rate $= 14\%$; semi-annual coupons; par $= \$1,000$; maturity $= 7$ years; $YTM = 16\%$ (APR).

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Valuing Semi-Annual Bonds

Example

Price the following bond: coupon rate $= 14\%$; semi-annual coupons; par $= \$1,000$; maturity $= 7$ years; $YTM = 16\%$ (APR).

Answer

$$\begin{aligned} BV &= \frac{(0.14 \times 1,000)}{2}\left[\frac{1-\frac{1}{(1+0.16/2)^{2\times 7}}}{0.16/2}\right]+\frac{1,000}{(1+0.16/2)^{2\times 7}} \\ &= 70 \times 8.2442 + 340.4610 \\ &= 917.56 \end{aligned}$$

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Interest Rate Risk

Price Risk

- Change in price due to changes in interest rates
- Long-term bonds have more price risk than short-term bonds (i.e., longer durations)
- Low coupon rate bonds have more price risk than high coupon rate bonds.

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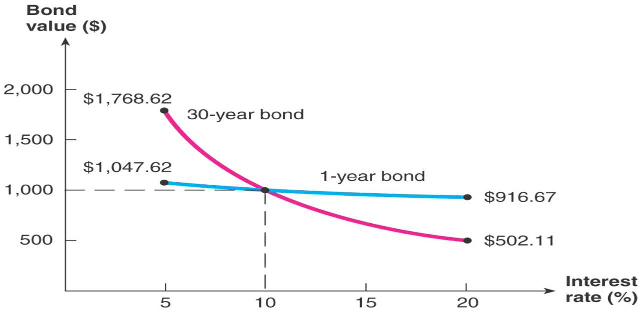
Reinvestment Risk

- Uncertainty concerning rates at which cash flows can be reinvested
- Short-term bonds have more reinvestment rate risk than long-term bonds.
- High coupon rate bonds have more reinvestment rate risk than low coupon rate bonds.

Interest Rate Risk
(cont'd)



Value of a Bond with a 10 Percent Coupon Rate for Different Interest Rates and Maturities

	Time to Maturity	
Interest Rate	1 Year	30 Years
5%	$1,047.62	$1,768.62
10	1,000.00	1,000.00
15	956.52	671.70
20	916.67	502.11

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Computing Yield-to-Maturity

Example

Consider a bond with a 10% annual coupon rate, 15 years to maturity and a par value of $1000. The current price is $928.09. What is the yield-to-maturity?

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You can use a financial calculator to solve the equation.

Alternatively, use the 'trial-and-error' method.

$$\$928.09 = (0.10 \times \$1,000)\left[\frac{1 - \frac{1}{(1+YTM)^{15}}}{YTM}\right] + \frac{\$1,000}{(1+YTM)^{15}}$$

Because the bond price < par value, YTM > coupon rate. This is a good starting point!
Try $YTM = 10.5\%$, we get $BV = \$963.03$ (Too high)
Try $YTM = 11.5\%$, we get $BV = \$895.05$ (Too low)
Try $YTM = 11\%$, we get $BV = \$928.09$ (Spot on!)

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Answer: 11%

Zero Coupon Bonds

- Make no periodic interest payments (coupon rate $= 0\%$)
- Entire yield-to-maturity comes from the difference between the purchase price and the par value (capital gains)
- Cannot sell for more than par value
- Sometimes called zeroes, or deep discount bonds
- Treasury Bills and U.S. Savings bonds are good examples of zeroes

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Valuing Zero Coupon Bonds

Example

A zero coupon bond with a maturity date of one year from today is currently traded at a YTM of 5%. What is the price of this bond if the face value is $1,000?

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Example

A zero coupon bond with a maturity date of one year from today is currently traded at a YTM of 5%. What is the price of this bond if the face value is \$1,000?

Answer:

On the maturity date, the bond pays \$1,000. There is no interim cash flow between today and the maturity date.

$$
\begin{aligned}
BV &= \frac{F}{(1+YTM)^t} \\
&= \frac{\$1,000}{(1.05)^1} \\
&= \$952.38
\end{aligned}
$$

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Bond Ratings

Investment Quality

High Grade

- Moody's Aaa and S&P AAA – capacity to pay is extremely strong (Microsoft, Exxon Mobil, and Johnson & Johnson, for example)
- Moody's Aa and S&P AA – capacity to pay is very strong (e.g., HSBC)

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Medium Grade

- Moody's A and S&P A – capacity to pay is strong, but more susceptible to changes in circumstances (e.g., Bank of America)
- Moody's Baa and S&P BBB – capacity to pay is adequate, adverse conditions will have more impact on the firm's ability to pay

Bond Ratings

Investment Quality (cont'd)

Low Grade

- Moody's Ba, B, Caa and Ca
- S&P BB, B, CCC, CC
- Considered speculative with respect to capacity to pay. The "B" ratings are the lowest degree of speculation.

Very Low Grade

- Moody's C and S&P C – income bonds with no interest being paid
- Moody's D and S&P D – in default with principal and interest in arrears

Bond Ratings

Investment Quality (cont'd)

Rating			
Moody's	S&P	Fitch	Definitions
Aaa	AAA	AAA	Prime Maximum Safety
Aa1	AA-	AA-	High Grade High Quality
Aa2	AA	AA	
Aa3	AA-	AA-	
A1	A+	A+	Upper Medium Grade
A2	A	A	
A3	A-	A-	
Baa1	BBB+	BBB+	Lower Medium Grade
Baa2	BBB	BBB	
Baa3	BBB-	BBB-	
Ba1	BB+	BB+	Non Investment Grade
Ba2	BB	BB	Speculative
Ba3	BB-	BB-	
B1	B-	B-	Highly Speculative
B2	B	B	
B3	B-	B-	
Caa1	CCC+	CCC	Substantial Risk
Caa2	CCC	—	In Poor Standing
Caa3	CCC-	—	
Ca	—	—	Extremely Speculative
C	—	—	May be in Default
—	—	DDD	Default
—	—	DD	—
—	D	D	

Government Bonds

- Treasury Securities = Federal government debt
- Treasury Bills (T-bills)
  - Pure discount bonds
    - Original maturity of one year or less
  - Treasury notes
    - Coupon debt
    - Original maturity between one and ten years
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- Municipal Securities
  - Debt of state and local governments
  - Varying degrees of default risk, rated similar to corporate debt
  - Interest received is tax-exempt

After-Tax Returns

A taxable bond has a yield of 8% and a municipal bond has a yield of 6%. If you are in a 40% tax bracket, which bond do you prefer?

$8\%\,(1 - 0.40) = 4.8\%$

The after-tax return on the corporate bond is 4.8%, compared to a 6% return on the municipal.

After-Tax Returns

- A taxable bond has a yield of 8% and a municipal bond has a yield of 6%. If you are in a 40% tax bracket, which bond do you prefer?
  - $8\% \, (1 - 0.40) = 4.8\%$
  - The after-tax return on the corporate bond is 4.8%, compared to a 6% return on the municipal.
- At what tax rate would you be indifferent between the two bonds?
  - $8\% \, (1 - T) = 6\%$
  - $T = 25\%$

Other Bond Types

Floating rate notes (FRN or floaters)

Income bonds

Convertible bonds

Puttable bonds

Callable bonds

Inflation and Interest Rates

Real rate of interest = change in purchasing power

Nominal rate of interest

= quoted rate of interest
= change in purchasing power and inflation

The ex ante nominal rate of interest includes our desired real rate of return plus an adjustment for expected inflation

Inflation and Interest Rates

- Real rate of interest = change in purchasing power
- Nominal rate of interest
  - = quoted rate of interest
  - = change in purchasing power and inflation
- The ex ante nominal rate of interest includes our desired real rate of return plus an adjustment for expected inflation
- The Fisher Effect defines the relationship between real rates, nominal rates and inflation:

$$\begin{aligned} (1+R) &= (1+r)(1+\pi) \\ R &\approx r+\pi \end{aligned}$$

where R = nominal rate (quoted rate), r = real rate and π = expected inflation rate.

Example

If we require a 10% real return and we expect inflation to be 8%, what is the nominal rate?

- If we require a 10% real return and we expect inflation to be 8%, what is the nominal rate?
- Answer:
  - $R = (1.1)(1.08) - 1 = 0.188 = 18.8\%$
  - Approximation: $R \approx 10\% + 8\% = 18\%$
  - Because the real return and expected inflation are relatively high, there is significant difference between the actual Fisher Effect and the approximation.

Term Structure of Interest Rates

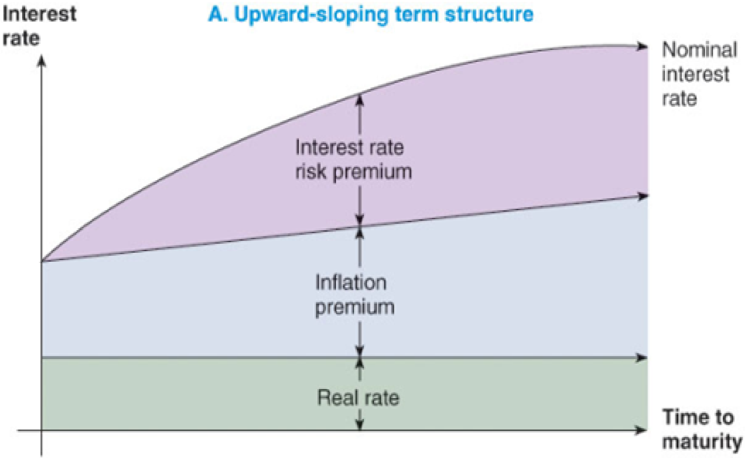
Term structure: The relationship between time to maturity and yields, all else equal

The effect of default risk, different coupons, etc. has been removed.

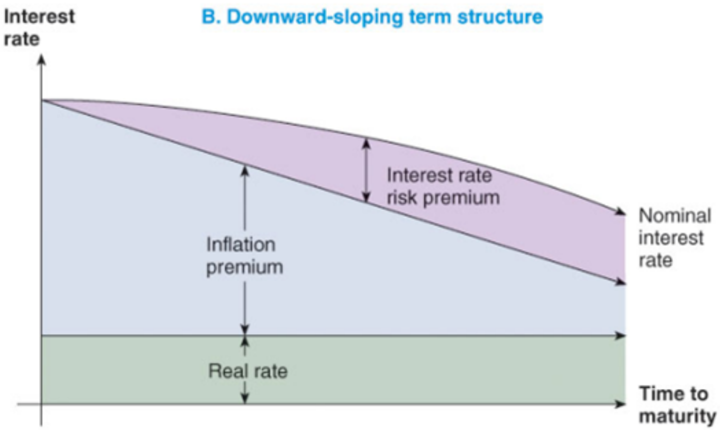
Yield curve: Graphical representation of the term structure

Normal $=$ upward-sloping $\Rightarrow$ long-term rates $>$ short-term rates
Inverted $=$ downward-sloping $\Rightarrow$ long-term rates $<$ short-term rates

Upward-Sloping Yield Curve



Downward-Sloping Yield Curve



Summary

- Pricing straight bonds are simple: find the sum of PVs of all the future coupon payments and the par value.
- Bonds with no periodic coupon payments are called "zeroes".
- To price a zero coupon bond, simply calculate the PV of the par value.
- Bondholders face two types of risk:
  - Price risk: change in price due to changes in interest rates
  - Reinvestment risk: uncertainty concerning rates at which cash flows can be reinvested
- Nominal interest rate = real interest rate + expected inflation rate
- A yield curve shows the level of nominal rate as a function of the time to maturity.