# Introduction to Finance (N11119)

Finance Tutor London

Lecture 4: Discounted Cash Flow Valuation (Part II)

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# Key Concepts and Skills

- Be able to find the interest rate on a loan
- Understand how interest rates are quoted
- Understand how loans are amortized or paid off

# Recall from the last lecture ...

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**An annuity** – a finite set of level sequential cash flows.

- If the first payment occurs at the end of the period, it is called an *ordinary annuity*.
- If the first payment occurs at the beginning of the period, it is called an *annuity due*.

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**A perpetuity –** is a perpetual annuity, or a set of level never-ending sequential cash flows, with the first cash flow occurring one period from now.

# Recall from the last lecture ...
(cont'd)

Perpetuity

$$PV = \frac{PMT}{r}$$

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Annuity

$$PV = PMT\left[\frac{1-\frac{1}{(1+r)^t}}{r}\right]$$

$$FV = PMT\left[\frac{(1+r)^t-1}{r}\right]$$

The term $\left[\frac{1-\left[1/(1+r)^t\right]}{r}\right]$ is called the *present value annuity factor*.
The term $\left[(1+r)^t-1\right]/r$ is called the *future value annuity factor*.

# Finding the Number of Payments

## Example I

You have \$1,000 outstanding balance on your credit card. You plan to pay \$20 month minimum. Rate = 1.5% per month. How many months before the balance is paid off?

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# Finding the Number of Payments

## Example I (cont'd)

Answer:

From the formula:

$$PV = PMT\left[\frac{1-\frac{1}{(1+r)^t}}{r}\right]$$

Plug in the known values and solve for $t$, we have:

$$\begin{aligned}
\$1,000 &= \$20\left[\frac{1-\frac{1}{(1.015)^t}}{0.015}\right] \\
(1.015)^t &= 4 \\
\ln(1.015)^t &= \ln(4) \\
t \times \ln(1.015) &= \ln(4) \\
t &= \frac{\ln(4)}{\ln(1.015)} \\
&= 93.111 \text{ months}
\end{aligned}$$

$93.111/12 = 7.76$ years $=$ 7 years and 9 months approx.

Important: Interest rate and time period must match!

# Finding the Number of Payments

## Example II

Suppose you borrow $2,000 at 5% and you are going to make annual payments of $732.42. How long before you pay off the loan?

# Finding the Number of Payments

## Example II (cont'd)

Answer:

From the formula:

$$PV = PMT\left[\frac{1-\frac{1}{(1+r)^t}}{r}\right]$$

Plug in the known values and solve for $t$, we have:

$$\begin{aligned}
\$2,000 &= \$732.42\left[\frac{1-\frac{1}{(1.05)^t}}{0.05}\right] \\
(1.05)^t &= 1.1581 \\
\ln(1.05)^t &= \ln(1.1581) \\
t \times \ln(1.05) &= \ln(1.1581) \\
t &= \frac{\ln(1.1581)}{\ln(1.05)} \\
&= 3.008 \text{ years}
\end{aligned}$$

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# Finding the Rate

## Example I

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Finding the rate by solving the PV equation is very difficult. It involves 'trial and error'.

$$PV = PMT\left[\frac{1-\frac{1}{(1+r)^t}}{r}\right]$$

$10000 = 207.58\left[\frac{1-\frac{1}{(1+r)^{60}}}{r}\right]$

Answer $r = 0.75\%$ per month

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# Annuity Due

## Example I

You are saving for a new house and you put $10,000 per year in an account paying 8%. The first payment is made today. How much will you have at the end of 3 years?

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## Example I

You are saving for a new house and you put $10,000 per year in an account paying 8%. The first payment is made today. How much will you have at the end of 3 years?

Answer:

$$\begin{aligned} FV &= PMT\left[\frac{(1+r)^t - 1}{r}\right](1+r) \\ &= 10,000\left[\frac{(1.08)^3 - 1}{0.08}\right](1.08) \\ &= \$35,061.12 \end{aligned}$$

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Notice that if we multiply the formula for the FV of an ordinary annuity

$$FV = PMT\left[\frac{(1+r)^t - 1}{r}\right]$$

by $(1+r)$, we get

$$FV = PMT\left[\frac{(1+r)^t - 1}{r}\right](1+r)$$

which is the formula for the FV of an annuity due. *Why?*

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## Example I (cont'd)

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which is the formula for the FV of an annuity due. *Why?*

Cash flows of an ordinary annuity occur at the end of each period.

For an annuity due, they occurs at the beginning of each period.

Each cash flow gets one extra compounding period.

# Perpetuity

## Example I

Perpetuity formula:

$$PV = \frac{PMT}{r}$$

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You are considering preferred stock that pays a quarterly dividend of \$1.50. If your desired return is 3% per quarter, how much would you be willing to pay?

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Answer:

$$\begin{aligned} PV &= \frac{PMT}{r} \\ &= \frac{\$1.50}{0.03} \\ &= \$50 \end{aligned}$$

Important: Interest rate and time period must match!

# Interest Rates

Effective Annual Rate (EAR)

- The interest rate expressed as if it were compounded once per year.
- Used to compare two alternative investments with different compounding periods

Annual Percentage Rate (APR) "Nominal"

- The annual rate quoted by law
- APR = periodic rate $\times$ number of periods per year
- Periodic rate = APR / periods per year

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- The annual rate quoted by law
- APR $=$ periodic rate $\times$ number of periods per year
- Periodic rate $=$ APR / periods per year

You *always* need to make sure that the interest rate and the time period match.

- Annual periods $\rightarrow$ annual rate.
- Monthly periods $\rightarrow$ monthly rate.

# Interest Rates
(cont'd)

Recall (from Lecture 2) that if the interest is compounded more than once a year:

$$FV = PV\left(1 + \frac{r}{n}\right)^{nt}$$

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# Interest Rates

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Recall (from Lecture 2) that if the interest is compounded more than once a year:

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If we set $PV = 1$ and $t = 1$, i.e., we want to find the FV at the end of a one-year period of \$1:

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(cont'd)

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The interest that we earn after this one-year period is therefore:

$$\begin{aligned}\text{Interest} &= FV - 1\\ &= \left(1+\frac{r}{n}\right)^{n} - 1\\ &= EAR\end{aligned}$$

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# Interest Rates
(cont'd)

The relationship between EAR and APR:

$$1 + EAR = \left[1 + \frac{APR}{n}\right]^n$$

EAR formula:

$$EAR = \left[1 + \frac{APR}{n}\right]^n - 1$$

APR formula:

$$APR = n\left[(1 + EAR)^{1/n} - 1\right]$$

note the n is the number of periods in a year

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# Interest Rates

## Example I

Which savings accounts should you choose:

5.25% annual interest rate with daily compounding.
5.30% annual interest rate with semiannual compounding.

# Interest Rates

## Example I (cont'd)

First account:

$$\begin{aligned} EAR &= \left[1 + \frac{0.0525}{365}\right]^{365} - 1 \\ &= 5.39\% \end{aligned}$$

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# Interest Rates

## Example I (cont'd)

First account:

$$EAR = \left[1 + \frac{0.0525}{365}\right]^{365} - 1 = 5.39\%$$

Second account:

$$EAR = \left[1 + \frac{0.053}{2}\right]^{2} - 1 = 5.37\%$$

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# Interest Rates

## Example II

Suppose you want to earn an effective rate of 12% and you are looking at an account that compounds on a monthly basis. What APR must they pay?

Suppose you want to earn an effective rate of 12% and you are looking at an account that compounds on a monthly basis. What APR must they pay?

Answer:

$$\begin{aligned} APR &= n\left[(1+EAR)^{1/n}-1\right] \\ &= 12\left[(1+0.12)^{1/12}-1\right] \\ &= 11.39\% \end{aligned}$$

# Pure Discount Loans

Treasury bills are excellent examples of pure discount loans.

- Principal amount is repaid at some future date
- No periodic interest payments

Other pure discount instruments:

- Certificates of Deposit
- Commercial Papers

# Pure Discount Loans

## Example

If a T-bill promises to repay $10,000 in 12 months and the market interest rate is 7 percent, how much will the bill sell for in the market?

# Pure Discount Loans

## Example

If a T-bill promises to repay $10,000 in 12 months and the market interest rate is 7 percent, how much will the bill sell for in the market?

Answer:

$$\begin{aligned} PV &= FV\,(1+r)^{-t} \\ &= \$10,000\,(1.07)^{-1} \\ & \$9,345.79 \end{aligned}$$

# Amortised Loan with Fixed Payment

## Example

Each payment covers the interest expense plus reduced principal.

Consider a 4-year loan with annual payments. The interest rate is 8% and the principal amount is \$5000. What is the annual payment?

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# Amortised Loan with Fixed Payment

## Example

Each payment covers the interest expense plus reduced principal.

Consider a 4-year loan with annual payments. The interest rate is 8% and the principal amount is \$5000. What is the annual payment?

Answer: From the formula for an ordinary annuity:

$$PV = PMT\left[\frac{1-\frac{1}{(1+r)^t}}{r}\right]$$

Solve for PMT

$$PMT = PV\left[\frac{r}{1-\frac{1}{(1+r)^t}}\right]$$

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# Amortised Loan with Fixed Payment

## Example (cont'd)

Plug in all the known values:

$$\begin{aligned} PMT &= \$5,000\left[\frac{0.08}{1-\frac{1}{(1+0.08)^4}}\right] \\ &= \$1,509.60 \end{aligned}$$

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# Amortised Loan with Fixed Payment

## Example (cont'd)

Plug in all the known values:

$$PMT = \$5,000\left[\frac{0.08}{1-\frac{1}{(1+0.08)^4}}\right] = \$1,509.60$$

The level annual payment is \$1,509.60.

Now let's take a look at the decomposition of this payment in each of the four years.

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# Amortised Loan with Fixed Payment

## Example (cont'd)

| year | beginning | total pmt | interest paid | principal paid | ending |
|---|---|---|---|---|---|
| 1 | 5000.00 | 1509.60 | 400.00 | 1109.60 | 3890.40 |
| 2 | 3890.40 | 1509.60 | 311.23 | 1198.37 | 2692.03 |
| 3 | 2692.03 | 1509.60 | 215.36 | 1294.24 | 1397.79 |
| 4 | 1397.79 | 1509.60 | 111.82 | 1397.78 | |
| total | | | 1038.42 | 5000.00 | |

Interest paid $=$ beginning balance $\times$ rate (8%)

Principal paid $=$ total payment $-$ interest paid

Ending balance $=$ beginning balance $-$ principal paid

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# Summary

- Treat each cash flow one-by-one if they are different.
- If cash flows are equal and finite, use the annuity formula. Pay attention to whether the cash flows occur at the end or the beginning of each period.
- If cash flows are equal and infinite, use the perpetuity formula.
- An amortising loan is a loan where the principal of the loan is paid down over the life of the loan according to some amortisation schedule, typically through equal payments.
    - During the early months of the loan, most of the payment will likely go toward interest.
    - Payment to principal gradually increases and interest payment drops, even though the payment amount is the same.