Introduction to Finance (N11119)

Finance Tutor London

Lecture 3: Discounted Cash Flow Valuation (Part I)

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Key Concepts and Skills

Be able to compute the future value of multiple cash flows

Be able to compute the present value of multiple cash flows

Be able to compute the present value and the future value of an ordinary annuity

Be able to compute loan payments

Multiple Cash Flows

In reality, most investment have multiple cash flows, for example:

- Bonds
- Stocks
- Derivatives such as credit default swaps (CDs)
- Investment projects

Future Value: Multiple Cash Flows

Example I

You think you will be able to deposit $4,000 at the end of each of the next three years in a bank account paying 8 percent interest. You currently have $7,000 in the account. How much will you have in 3 years?

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Future Value: Multiple Cash Flows

Example I

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Answer: Find the value at year 3 of each cash flow and add them together.

Year 0: $FV = \$7,000\,(1.08)^3 = \$8,817.98$
Year 1: $FV = \$4,000\,(1.08)^2 = \$4,665.60$
Year 2: $FV = \$4,000\,(1.08)^1 = \$4,320.00$
Year 3: $V = \$4,000$
Total value in 3 years: $8,817.98 + 4,665.60 + 4,320.00 + 4,000 = \$21,803.58$

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Assume you stop making the deposit, how much will you have in 4 years?

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Total value in 3 years: $8,817.98 + 4,665.60 + 4,320.00 + 4,000 = \$21,803.58$

Assume you stop making the deposit, how much will you have in 4 years?

Answer:

Total value in 4 years: $\$21,803.58\,(1.08)^1 = \$23,547.87$

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Future Value: Multiple Cash Flows

Example II

If you deposit \$100 in one year, \$200 in two years and \$300 in three years. How much will you have in three years at 7 percent interest?

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Future Value: Multiple Cash Flows

Example II

If you deposit \$100 in one year, \$200 in two years and \$300 in three years. How much will you have in three years at 7 percent interest?

Answer:

Year 1: $FV = \$100\,(1.07)^2 = \$114.49$
Year 2: $FV = \$200\,(1.07)^1 = \$214.00$
Year 3: $V = \$300$
Total value in 3 years: $114.49 + 214.00 + 300.00 = \$628.49$

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Future Value: Multiple Cash Flows

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How much in five years will you have if you don't add additional amounts?

Future Value: Multiple Cash Flows

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Year 3: $V = \$300$
Total value in 3 years: $114.49 + 214.00 + 300.00 = \$628.49$

How much in five years will you have if you don't add additional amounts?

Answer:

Total value of 5 years: $\$628.49\,(1.07)^2 = \$719.56$

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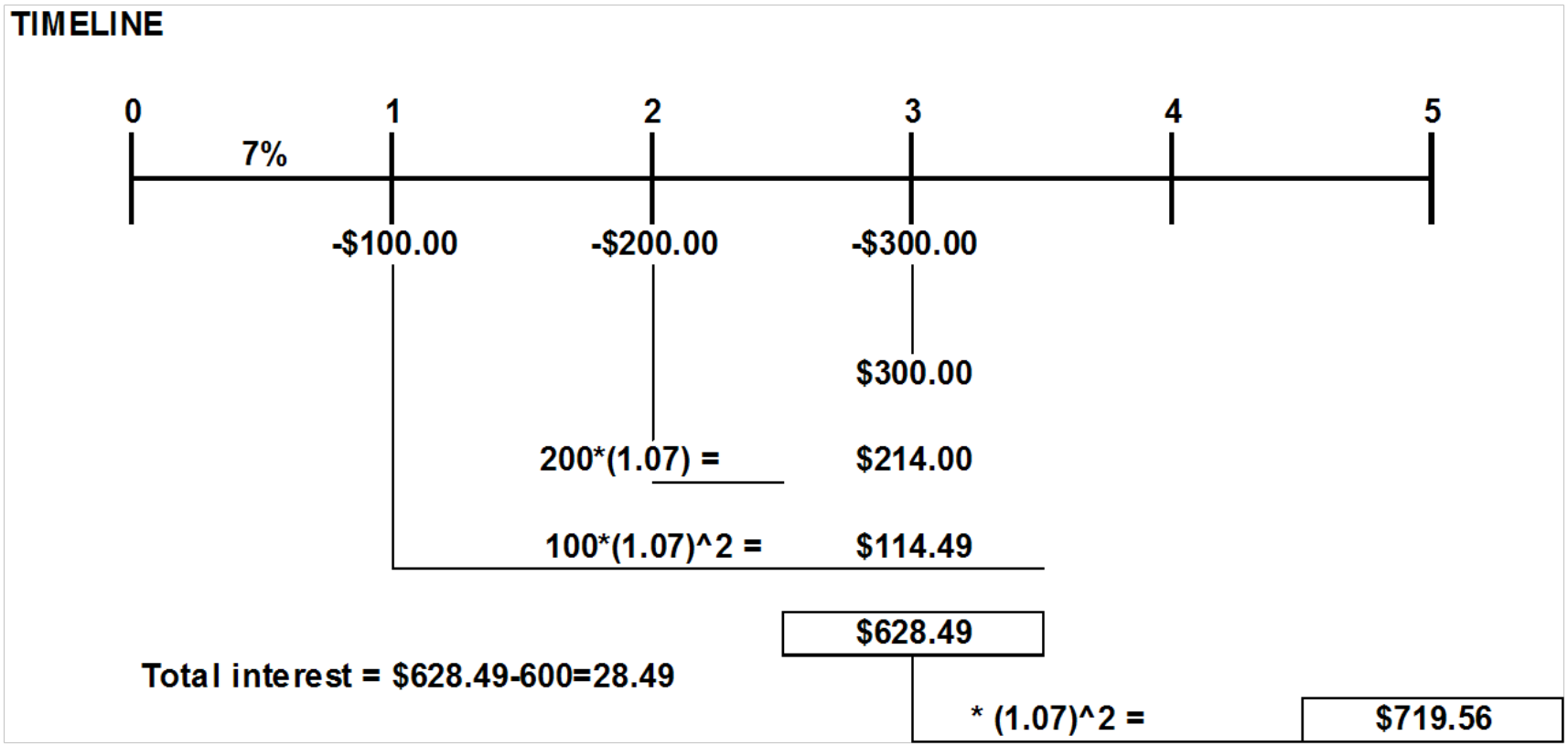
Future Value: Multiple Cash Flows

Example II: Time Line



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Future Value: Multiple Cash Flows

Example III

Suppose you plan to deposit \$100 into an account in one year and \$300 into the account in three years. How much will be in the account in five years if the interest rate is 8%?

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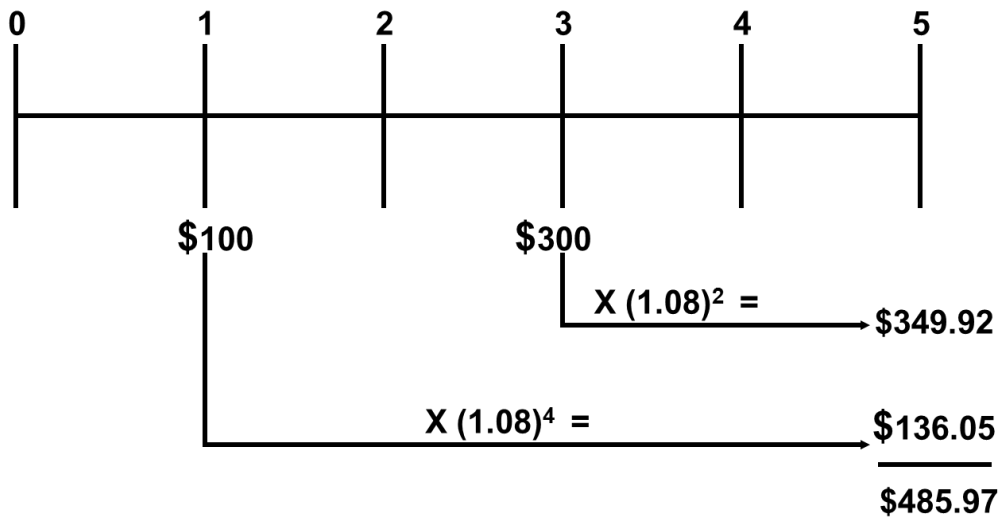
Future Value: Multiple Cash Flows

Example III

Suppose you plan to deposit \$100 into an account in one year and \$300 into the account in three years. How much will be in the account in five years if the interest rate is 8%?

Answer:

$$FV = \$100\,(1.08)^4 + \$300\,(1.08)^2 = 136.05 + 349.92 = \$485.97$$



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Present Value: Multiple Cash Flows

Example I

You are offered an investment that will pay: \$200 in year 1, \$400 the next year, \$600 the following year, and \$800 at the end of the 4th year. You can earn 12% on similar investments. What is the most you should pay for this one?

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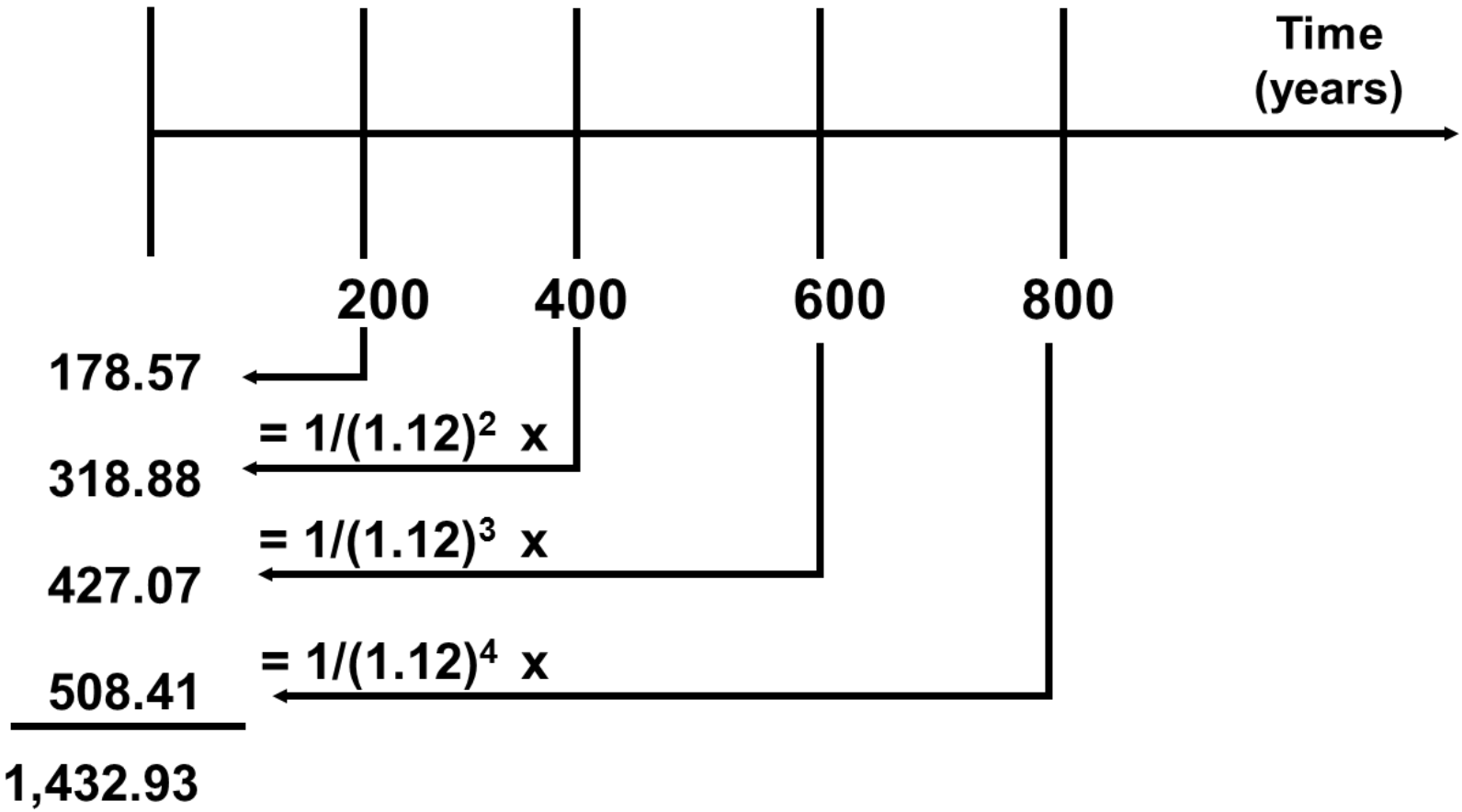
Answer: Find the PV of each cash flow and add them:

$$\text{Year 1: } PV = \$200\,(1.12)^{-1} = \$178.57$$
$$\text{Year 2: } PV = \$400\,(1.12)^{-2} = \$318.88$$
$$\text{Year 3: } PV = \$600\,(1.12)^{-3} = \$427.07$$
$$\text{Year 4: } PV = \$800\,(1.12)^{-4} = \$508.41$$
$$\text{Total } PV = 178.57 + 318.88 + 427.07 + 508.41 = \$1,432.93$$

Present Value: Multiple Cash Flows

Example I: Time Line



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Present Value: Multiple Cash Flows

Example II

You are considering an investment that will pay you \$1,000 in one year, \$2,000 in two years and \$3,000 in three years. If you want to earn 10% on your money, how much would you be willing to pay?

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Present Value: Multiple Cash Flows

Example II

You are considering an investment that will pay you \$1,000 in one year, \$2,000 in two years and \$3,000 in three years. If you want to earn 10% on your money, how much would you be willing to pay?

Answer: Find the PV of each cash flow and add them:

$$\text{Year 1: } PV = \$1,000\,(1.10)^{-1} = \$909.09$$
$$\text{Year 2: } PV = \$2,000\,(1.10)^{-2} = \$1,652.89$$
$$\text{Year 3: } PV = \$3,000\,(1.10)^{-3} = \$2,253.94$$
$$\text{Total } PV = 909.09 + 1,652.89 + 2,253.94 = \$4,815.92$$

Present Value: Multiple Cash Flows

Example III

Your broker calls you and tells you that he has this great investment opportunity. If you invest $100 today, you will receive $40 in one year and $75 in two years. If you require a 15% return on investments of this risk, should you take the investment?

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Present Value: Multiple Cash Flows

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Answer: Find the PV of each cash flow and add them:

$$\text{Year 1: } PV = \$40\,(1.15)^{-1} = \$34.78$$
$$\text{Year 2: } PV = \$75\,(1.15)^{-2} = \$56.71$$
$$\text{Total } PV = 34.78 + 56.71 = \$91.49$$

No – the broker is charging more than you would be willing to pay.

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Present Value: Multiple Cash Flows

Example IV: Saving for Retirement

You are offered the opportunity to put some money away for retirement. You will receive five annual payments of $25,000 each beginning in 40 years. How much would you be willing to invest today if you desire an interest rate of 12%?

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Present Value: Multiple Cash Flows

Example IV: Saving for Retirement

You are offered the opportunity to put some money away for retirement. You will receive five annual payments of $25,000 each beginning in 40 years. How much would you be willing to invest today if you desire an interest rate of 12%?

Answer: Find the values of each cash flow in year 40 and add them:

Year 40: $V = \$25,000$
Year 41: $PV = \$25,000\,(1.12)^{-1} = \$22,321.43$
Year 42: $PV = \$25,000\,(1.12)^{-2} = \$19,929.85$
Year 43: $PV = \$25,000\,(1.12)^{-3} = \$17,794.51$
Year 44: $PV = \$25,000\,(1.12)^{-4} = \$15,887.95$
Total value in Year 40: $\$100,933.74$

Present Value: Multiple Cash Flows

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Year 43: $PV = \$25,000\,(1.12)^{-3} = \$17,794.51$
Year 44: $PV = \$25,000\,(1.12)^{-4} = \$15,887.95$
Total value in Year 40: $\$100,933.74$

Find the PV of the value calculated for Year 40:

Total $PV = \$100,933.74\,(1.12)^{-40} = \$1,084.71$

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Present Value & Future Values: Multiple Cash Flows

Example I

Suppose you are looking at the following possible cash flows: Year 1 CF = \$100; Years 2 and 3 CFs = \$200; Years 4 and 5 CFs = \$300. The required discount rate is 7% What is the value of the CFs at Year 5?

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Present Value & Future Values: Multiple Cash Flows

Example I

Suppose you are looking at the following possible cash flows: Year 1 CF = \$100; Years 2 and 3 CFs = \$200; Years 4 and 5 CFs = \$300. The required discount rate is 7% What is the value of the CFs at Year 5?

Answer: The value of the cash flows at Year 5

$$\begin{aligned} CF_5 &= 100\,(1.07)^4 + 200\,(1.07)^3 + 200\,(1.07)^2 + 300\,(1.07) + 300 \\ &= \$1,226.07 \end{aligned}$$

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What is the value of the CFs today?

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What is the value of the CFs today?

Answer: The value of the cash flows today (Year 0)

$$\begin{aligned} CF_0 &= 100\,(1.07)^{-1} + 200\,(1.07)^{-2} + 200\,(1.07)^{-3} \\ &\quad +300\,(1.07)^{-4} + 300\,(1.07)^{-5} \\ &= \$874.17 \end{aligned}$$

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Present Value & Future Values: Multiple Cash Flows

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Suppose you are looking at the following possible cash flows: Year 1 CF = \$100; Years 2 and 3 CFs = \$200; Years 4 and 5 CFs = \$300. The required discount rate is 7% What is the value of the CFs at Year 5?

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Answer: The value of the cash flows today (Year 0)

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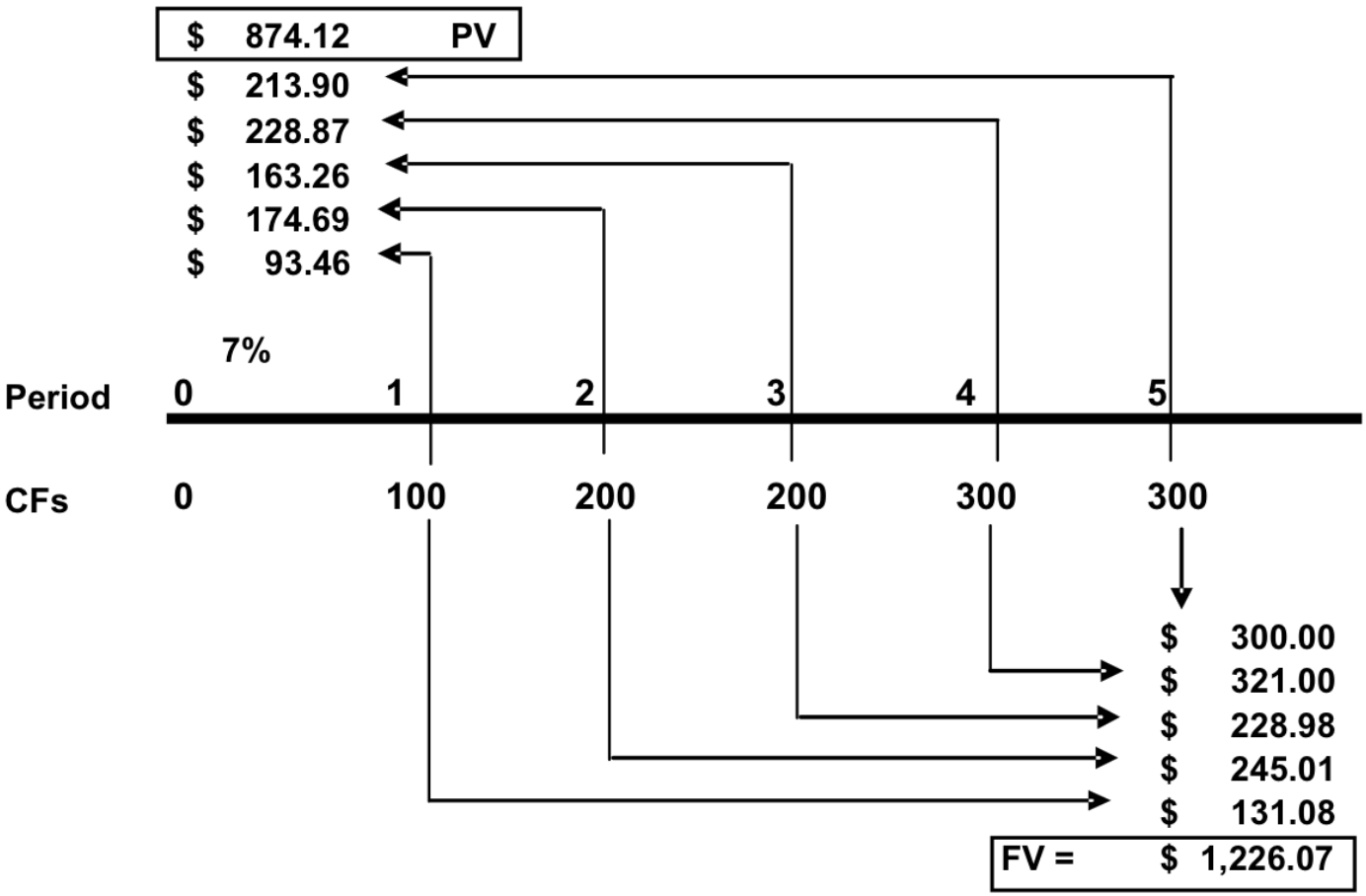
Note that:

$$\$874.17\,(1.07)^5 = \$1,226.07$$

Present Value & Future Values: Multiple Cash Flows

Example I: Time Line



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Annuity & Perpetuity

An annuity – a finite set of level sequential cash flows.

If the first payment occurs at the end of the period, it is called an *ordinary annuity*.
If the first payment occurs at the beginning of the period, it is called an *annuity due*.

Annuity & Perpetuity

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A perpetuity – is a perpetual annuity, or a set of level never-ending sequential cash flows, with the first cash flow occurring one period from now.

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Annuity & Perpetuity

Basic Formulas

Perpetuity

$$PV = \frac{PMT}{r}$$

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Annuity & Perpetuity

Basic Formulas

Perpetuity

$$PV = \frac{PMT}{r}$$

Annuity

$$PV = PMT \left[\frac{1 - \frac{1}{(1+r)^t}}{r} \right]$$

$$FV = PMT \left[\frac{(1+r)^t - 1}{r} \right]$$

The term $\left[\frac{1-\left[1/(1+r)^t\right]}{r} \right]$ is called the *present value annuity factor*.

The term $\left[(1+r)^t - 1\right]/r$ is called the *future value annuity factor*.

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Annuity

Example I

Suppose you begin saving for your retirement by depositing \$2,000 per year in an IRA.[1] If the interest rate is 7.5%, how much will you have in 40 years?

[1] IRA = Individual Retirement Account

Annuity

Example I

- Suppose you begin saving for your retirement by depositing \$2,000 per year in an IRA.[1] If the interest rate is 7.5%, how much will you have in 40 years?
- Answer:

$$\begin{aligned} FV &= PMT\left[\frac{(1+r)^t - 1}{r}\right] \\ &= \$2,000\left[\frac{(1.075)^{40} - 1}{0.075}\right] \\ &= \$454,513.04 \end{aligned}$$

[1] IRA = Individual Retirement Account

Annuity

Example II

Suppose your company's defined contribution (DC) retirement plan allows you to invest up to $20,000 per year. You plan to invest $20,000 per year in a stock index fund for the next 30 years.

Historically, this fund has earned 9 percent per year on average. Assuming that you can actually earn 9 percent per year, how much money will you have available for retirement after making the last payment?

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Annuity

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Historically, this fund has earned 9 percent per year on average. Assuming that you can actually earn 9 percent per year, how much money will you have available for retirement after making the last payment?

Answer:

$$\begin{aligned} FV &= \$20,000 \left[\frac{(1+0.09)^{30}-1}{0.09} \right] \\ &= \$2,726,150.77 \end{aligned}$$

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Annuity

Example III

You can afford \$632 per month. Going rate = 1% per month for 48 months. How much can you borrow?

Hint: You borrow money *today* so you need to compute the present value.

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Annuity

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You can afford \$632 per month. Going rate = 1% per month for 48 months. How much can you borrow?

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Answer:

$$\begin{aligned} PV &= \$632 \left[\frac{1 - \frac{1}{(1.01)^{48}}}{0.01} \right] \\ &= \$23,999.54 \end{aligned}$$

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Annuity

Example III

You can afford \$632 per month. Going rate = 1% per month for 48 months. How much can you borrow?

Hint: You borrow money *today* so you need to compute the present value.

Answer:

$$\begin{aligned} PV &= \$632 \left[\frac{1 - \frac{1}{(1.01)^{48}}}{0.01} \right] \\ &= \$23,999.54 \end{aligned}$$

Important: Interest rate and time period must match!

Annual periods $\Rightarrow$ annual rate
Monthly periods $\Rightarrow$ monthly rate

Annuity

Example IV

Suppose you win the Publishers Clearinghouse $10 million sweepstakes. The money is paid in equal annual installments of $333,333.33 over 30 years. If the appropriate discount rate is 5%, how much is the sweepstakes actually worth today?

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Annuity

Example IV

Suppose you win the Publishers Clearinghouse \$10 million sweepstakes. The money is paid in equal annual installments of \$333,333.33 over 30 years. If the appropriate discount rate is 5%, how much is the sweepstakes actually worth today?

Answer:

$$\begin{aligned} PV &= \$333,333.33 \left[\frac{1 - \frac{1}{(1.05)^{30}}}{0.05} \right] \\ &= \$5,124,150.29 \end{aligned}$$

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Finding the Payment

Example

Suppose you want to borrow \$20,000 for a new car. You can borrow at 8% per year, compounded monthly ($8/12 = .66667\%$ per month). If you take a 4 year loan, what is your monthly payment?

Finding the Payment

Example

Suppose you want to borrow \$20,000 for a new car. You can borrow at 8% per year, compounded monthly (8/12 = .66667% per month). If you take a 4 year loan, what is your monthly payment?

Answer:

From the formula:

$$PV = PMT \left[\frac{1 - \frac{1}{(1+r)^t}}{r} \right]$$

Plug in the known values and solve for PMT, we have:

$$\begin{aligned} \$20,000 &= PMT \left[\frac{1 - \frac{1}{(1.0066667)^{48}}}{0.0066667} \right] \\ PMT &= \$488.26 \end{aligned}$$

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Buying a House

Example

You are ready to buy a house and you have $20,000 for a down payment and closing costs.

Closing costs are estimated to be 4% of the loan value.

You have an annual after tax salary of $36,000.

The bank is willing to allow your monthly mortgage payment to be equal to 28% of your monthly income.

The interest rate on the loan is 6% per year with monthly compounding (.5% per month) for a 30-year fixed rate loan.

How much money will the bank loan you?

How much can you offer for the house?

Buying a House

Example (cont'd)

First, calculate the amount of loan based on your monthly repayment:

$$\text{Monthly income} = \$36,000/12 = \$3,000$$
$$\text{Maximum payment} = \$3,000 \times 0.28 = \$840$$

$$\begin{aligned} PV &= \$840 \left[\frac{1 - \frac{1}{(1.005)^{360}}}{0.005} \right] \\ &= \$140,105 \end{aligned}$$

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Buying a House

Example (cont'd)

Next, calculate the total price:

$$\text{Closing costs} = \$140,105 \times 0.04 = \$5,604$$
$$\text{Down payment} = \$20,000 - \$5,604 = \$14,396$$
$$\text{Total price} = \$140,105 + \$14,396 = \$154,501$$

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Summary

Present values and future values of multiple cash flows can be calculated by treating each cash flow as an individual cash flow.

Annuity – finite series of equal payments that occur at regular intervals

- If the first payment occurs at the end of the period, it is called an ordinary annuity
- If the first payment occurs at the beginning of the period, it is called an annuity due

Perpetuity – infinite series of equal payments.

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