Introduction to Finance (N11119)

Finance Tutor London

Lecture 2: The Time Value of Money

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Key Concepts and Skills

Be able to compute:

- The future value of an investment made today
- The present value of cash to be received at some future date
- The return on an investment
- The number of periods that equates a present value and a future value given an interest rate

Be able to solve time value of money problems using formulas

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Basic Definitions

Present Value (PV)

The current value of future cash flows discounted at the appropriate discount rate

Value at $t = 0$ on a time line

Future Value (FV)

The amount an investment is worth after one or more periods.

"Later" money on a time line

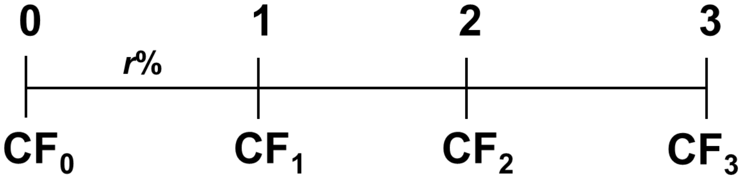
Basic Definitions
(cont'd)

- Interest rate (r)
  - Discount rate
  - Cost of capital
  - Opportunity cost of capital
  - Required return
- Terminology depends on usage

Time Line of Cash Flows

Tick marks at ends of periods

Time 0 is today;
Time 1 is the end of period 1



$+CF$ = cash inflow, $-CF$ = cash outflow and PMT = constant CF

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Present Values & Future Values: General Formula

The relationship between PV and FV:

$$FV = PV\,(1+r)^t$$

where FV = future value, PV = present value, r = period interest rate (expressed as a decimal) and t = number of periods.

The term $(1+r)^t$ is called "future value interest factor".

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Future Values for

Example I

Suppose you invest \$100 for one year at 10% per year. What is the future value in one year?

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Future Values

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Answer:

Interest $= 100 \times 10\% = 100\,(0.10) = \$10$
Value in one year $=$ principal $+$ interest $= 100 + 10 = \$110$
Using the formula: $FV = PV\,(1 + r)^t = 100\,(1 + 0.10)^1 = \$110$

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Suppose you leave the money in for another year. How much will you have two years from now?

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Suppose you leave the money in for another year. How much will you have two years from now?

Answer:

$$FV = 100\,(1.10)\,(1.10) = 100\,(1.10)^2 = \$121$$

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Effects of Compounding

Simple interest

- Interest earned only on the original principal

Compound interest

- Interest earned on principal and on interest received
- "Interest on interest" — interest earned on reinvestment of previous interest payments

Effects of Compounding
(cont'd)

Consider the previous example

- FV with simple interest $= 100 + 10 + 10 = \$120$ (At the end of year 1, you make a withdrawal of \$10.)
- FV with compound interest $= 100\,(1.10)^2 = \$121$ (You receive an extra \$1, which is the "interest on interest".)

The extra 1.00 comes from the interest of $0.10\,(10) = \$1.00$ earned on the first interest payment.

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Future Values

Example II

Suppose you invest the \$100 from the previous example for 5 years. How much would you have at the end of year 5?

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Answer:

$$FV = PV\,(1+r)^t$$
$$FV = 100\,(1.10)^5 = 100\,(1.6105) = \$161.05$$

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TABLE 4.1

Future value of $100 at 10 percent

Year	Beginning Amount	Interest Earned	Ending Amount
1	$100.00	$10.00	$110.00
2	110.00	11.00	121.00
3	121.00	12.10	133.10
4	133.10	13.31	146.41
5	146.41	14.64	161.05
	Total interest	$61.05	

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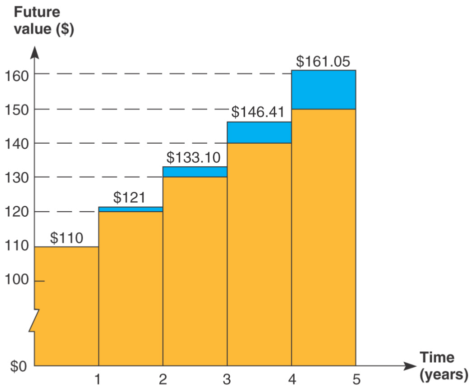
Future Values

Example II (cont'd)



Growth of $100 original amount at 10% per year. Blue shaded area represents the portion of the total that results from compounding of interest.

Compounding Frequency

Using the same example, suppose the interest is calculated every 6 months, then after 6 months:

$$FV = 100 + (100 \times 0.10/2) = \$105$$

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$$FV = 100 + (100 \times 0.10/2) = \$105$$

The first $5 of interest is capitalised and the loan is rolled over for another 6 months. So at the end of the 12 months:

$$FV = 105 + (105 \times 0.10/2) = \$110.25$$

We get an addition 25 cents if the interest is *semi-annually* compounded. Its *effective rate* is $10.25 \div 100 = 10.25\%$ instead of 10%.

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Compounding Frequency
(cont'd)

Suppose the interest is compounded quarterly, then after 3 months:

$$FV = 100 + (100 \times 0.10/4) = \$102.50$$

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Compounding Frequency
(cont'd)

Suppose the interest is compounded quarterly, then after 3 months:

$$FV = 100 + (100 \times 0.10/4) = \$102.50$$

The first $2.50 of interest is capitalised and the loan is rolled over for another 3 months. So at the end of month 6:

$$FV = 102.50 + (102.50 \times 0.10/4) = \$105.0625$$

Compounding Frequency
(cont'd)

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$$FV = 102.50 + (102.50 \times 0.10/4) = \$105.0625$$

The cumulative interest of $5.0625 is capitalised and the loan is rolled over for another 3 months. So at the end of month 9:

$$FV = 105.0625 + (105.0625 \times 0.10/4) = \$107.6891$$

Compounding Frequency
(cont'd)

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The cumulative interest of \$5.0625 is capitalised and the loan is rolled over for another 3 months. So at the end of month 9:

$$FV = 105.0625 + (105.0625 \times 0.10/4) = \$107.6891$$

The cumulative interest of \$7.6891 is capitalised and the loan is rolled over for another 3 months. So at the end of the 12 months:

$$FV = 107.6891 + (107.6891 \times 0.10/4) = \$110.3813$$

Compounding Frequency
(cont'd)

In general,

$$FV = PV\left(1 + {}^{r}/_{n}\right)^{nt}$$

where

n is the number of times the interest is compounded in a year
t is the number of years

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Future Values: Compounding Frequency
Example III

Suppose the interest is compounded quarterly. How much would you have if you invest \$100 for 5 years. ?

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Future Values: Compounding Frequency

Example III

Suppose the interest is compounded quarterly. How much would you have if you invest \$100 for 5 years. ?

Answer:

$$FV = PV\left(1 + {}^{r}/_{n}\right)^{nt}$$
$$FV = 100\left(1 + {}^{0.10}/_{4}\right)^{4\times5} = 100\,(1.6386) = 163.86$$

Compare to the amount of interest of \$61.05, you earn an extra $63.86 - 61.05 = \$2.81$ with quarterly compounding.

This number probably doesn't look like much. But if you invest \$1 million, the difference would be \$28,100!

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Compounding Frequency

Continuous Compounding

Suppose the interest is compound "continuously" (e.g., every second), how do we calculate the future value?

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From

$$FV = PV\left(1 + {}^{r}\!/_{n}\right)^{nt}$$

let $n \to \infty$.

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Compounding Frequency

Continuous Compounding

Suppose the interest is compound "continuously" (e.g., every second), how do we calculate the future value?

From

$$FV = PV\left(1 + {}^{r}/_{n}\right)^{nt}$$

let $n \to \infty$.

In the limit,

$$\lim_{n \to \infty} FV = PV \cdot e^{rt}$$

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Future Values: Compounding Frequency

Example IV

From our previous example, suppose the interest is compounded continuously. How much would you have if you invest $100 for 5 years. ?

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Future Values: Compounding Frequency

Example IV

From our previous example, suppose the interest is compounded continuously. How much would you have if you invest $100 for 5 years. ?

Answer:

$$FV = PV \cdot e^{rt}$$
$$FV = 100 \cdot e^{(0.10)(5)} = \$164.87$$

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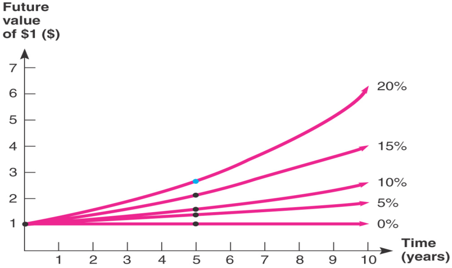
Future Values

Important Relationship

For a given interest rate: the longer the time period, the higher the future value.

For a given time period: the higher the interest rate, the larger the future value.



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Present Values

- The current value of future cash flows discounted at the appropriate discount rate
- Value at $t = 0$ on a time line
- Answers questions such as:
  - How much do I have to invest today to have some amount in the future?
  - What is the current value of an amount to be received in the future?

Present Values

The current value of future cash flows discounted at the appropriate discount rate

Value at $t = 0$ on a time line

Answers questions such as:

- How much do I have to invest today to have some amount in the future?
- What is the current value of an amount to be received in the future?

Why is it worth less than the future value?

- Opportunity cost
- Risk and uncertainty

Discount rate $= f\,(\text{time}, \text{risk})$, i.e., time and uncertainty.

Present Values

(cont'd)

From the future value formula,

$$FV = PV\,(1+r)^t$$

re-arrange to solve for PV

$$PV = \frac{FV}{(1+r)^t} = FV\,(1+r)^{-t}$$

"Discounting" = finding the present value of one or more future amounts

Finding the present value is called *discounting*, which is the reverse of compounding.

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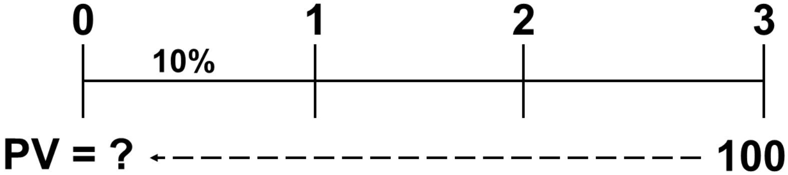
Present Values
(cont'd)



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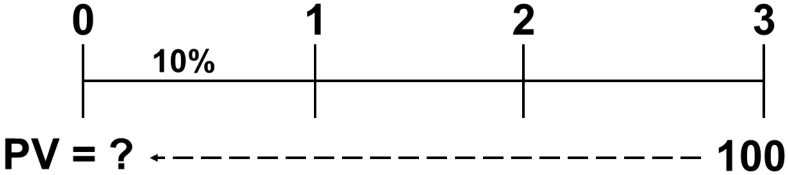
Present Values
(cont'd)



$$PV = FV\,(1+r)^{-t} = 100\,(1.10)^{-3} = \$75.13$$

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Present Values

Example I

Suppose you need $10,000 in one year for the down payment on a new car. If you can earn 7% annually, how much do you need to invest today?

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Present Values

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Suppose you need \$10,000 in one year for the down payment on a new car. If you can earn 7% annually, how much do you need to invest today?

Answer:

$$PV = FV\,(1+r)^{-t}$$
$$PV = 10,000\,(1.07)^{-1} = \$9,345.79$$

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Present Values

Example II

You want to begin saving for your daughter's university education and you estimate that she will need $150,000 in 17 years. If you feel confident that you can earn 8% per year, how much do you need to invest today?

Present Values

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Answer:

$$PV = FV\,(1+r)^{-t}$$
$$PV = 150,000\,(1.08)^{-17} = \$40,540.34$$

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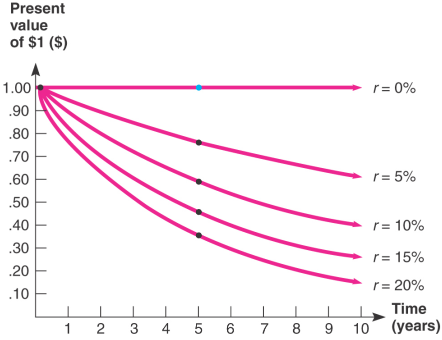
Present Values

Important Relationship

For a given interest rate: the longer the time period, the lower the present value.

For a given time period: the higher the interest rate, the smaller the present value.



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The Discount Rate

From the basic equation:

$$FV = PV\,(1 + r)^{t}$$

There are four parts to this equation.

- PV, FV, r and t
- Know any three, solve for the fourth

To find the *implied interest rate*, rearrange the equation and solve for r:

$$r = \left(\frac{FV}{PV}\right)^{1/t} - 1$$

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The Discount Rate

Example I

You are looking at an investment that will pay \$1,200 in 5 years if you invest \$1,000 today. What is the implied rate of interest?

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The Discount Rate

Example I

You are looking at an investment that will pay $1,200 in 5 years if you invest $1,000 today. What is the implied rate of interest?

Answer:

$$r = (1,200/1,000)^{1/5} - 1 = 0.03714 = 3.714\%$$

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The Discount Rate

Example II

Suppose you are offered an investment that will allow you to double your money in 6 years. You have \$10,000 to invest. What is the implied rate of interest?

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The Discount Rate

Example II

Suppose you are offered an investment that will allow you to double your money in 6 years. You have $10,000 to invest. What is the implied rate of interest?

Answer:

$$r = (20,000/10,000)^{1/6} - 1 = 0.1225 = 12.25\%$$

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The Number of Periods

Start with the basic equation:

$$FV = PV\left(1+r\right)^{t}$$

and solve for t.

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Start with the basic equation:

$$FV = PV\left(1 + r\right)^t$$

and solve for t.

$$t = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln\left(1 + r\right)}$$

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The Number of Periods

Example I

You want to purchase a new car and you are willing to pay \$20,000. If you can invest at 10% per year and you currently have \$15,000, how long will it be before you have enough money to pay cash for the car?

The Number of Periods

Example I

You want to purchase a new car and you are willing to pay \$20,000. If you can invest at 10% per year and you currently have \$15,000, how long will it be before you have enough money to pay cash for the car?

Answer:

$$t = \ln\left(\frac{FV}{PV}\right) / \ln(1+r) = \ln\left(\frac{20{,}000}{15{,}000}\right) / \ln(1+0.10) = 3.02 \text{ years}$$

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The Number of Periods

Example II

Suppose you want to buy some new furniture for your family room. You currently have \$500 and the furniture you want costs \$600. If you can earn 6%, how long will you have to wait if you don't add any additional money?

The Number of Periods

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Suppose you want to buy some new furniture for your family room. You currently have \$500 and the furniture you want costs \$600. If you can earn 6%, how long will you have to wait if you don't add any additional money?

Answer:

$$t = \ln\left(\frac{FV}{PV}\right) / \ln(1+r) = \ln\left(\frac{600}{500}\right) / \ln(1+0.06) = 3.13 \text{ years}$$

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Summary

For a given rate of interest

the longer the time period

the greater the future value
the smaller the present value

.

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Summary

For a given rate of interest

- the longer the time period
  - the greater the future value
  - the smaller the present value

For a given time period

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Summary

- For a given rate of interest
  - the longer the time period
    - the greater the future value
    - the smaller the present value
- For a given time period
  - the higher the interest rate
    - the greater the future value
    - the smaller the present value
- For a given rate of interest and a present value, the more frequent the interest is compounded, the greater the amount of interest and future value.

Summary

- For a given rate of interest
    - the longer the time period
        - the greater the future value
        - the smaller the present value
- For a given time period
    - the higher the interest rate
        - the greater the future value
        - the smaller the present value
- For a given rate of interest and a present value, the more frequent the interest is compounded, the greater the amount of interest and future value.
- For a given rate of interest and a future value, the more frequent the value (or cash flows) is (are) discounted, the smaller the amount of present value.