Valuation of bonds and stocks

SMN 224 – Corporate Finance

Week 4

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Outline

1. Definition of a Bond

- A bond is a legally binding agreement between a borrower and a lender in which the borrower (bond issuer) commits to repay the principal and interest at a certain date in the future
- In particular, the bond agreement specifies the:
  - Par (face) value
  - Coupon rate
  - Coupon payment
  - Maturity Date
- The ***yield to maturity*** is the required market interest rate on the bond

2. How to Value Bonds

- Primary Principle:
  - Value of financial securities = PV of expected future cash flows

- Bond value is, therefore, determined by the present value of the coupon payments and par value

- Interest rates (YTM) are inversely related to present (i.e., bond) values

The Bond Pricing Equation

Coupon payments

Principal repayment

$$\text{Bond Value} = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right] + \frac{FV}{(1+r)^T}$$

Remember: this is the formula for the present value of an annuity

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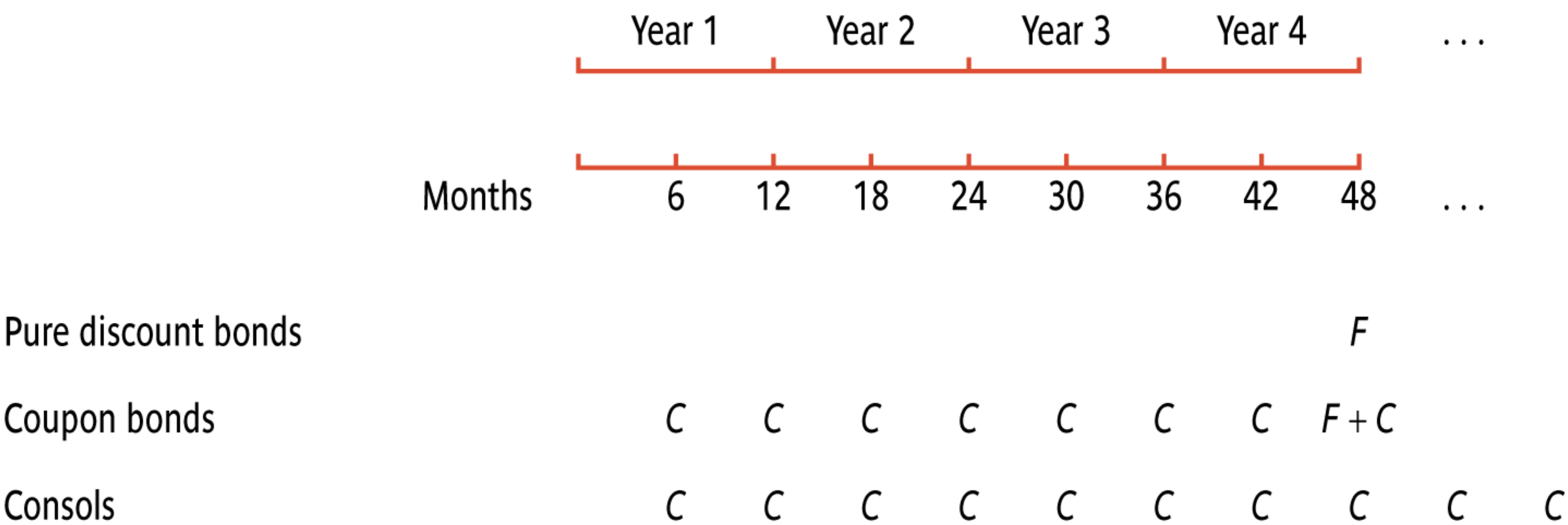
Three types of bonds

- Zero-coupon (pure discount) bonds
- Level bonds
- Consols



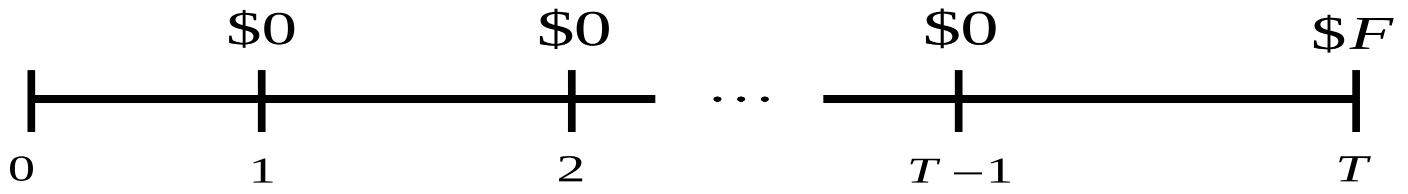
Pure Discount Bonds

- Make no periodic interest payments (coupon rate = 0%)
- The entire yield to maturity comes from the difference between the purchase price and the par value
- Cannot sell for more than par value
- Sometimes called *zeroes*, *deep discount bonds*, or *original issue discount bonds* (OIDs)
- Treasury Bills and principal-only Treasury strips are good examples of zeroes

Pure Discount Bonds

Information needed for valuing pure discount bonds:

- Time to maturity (*T*) = Maturity date - today's date
- Face value (*F*)
- Discount rate (*r*)

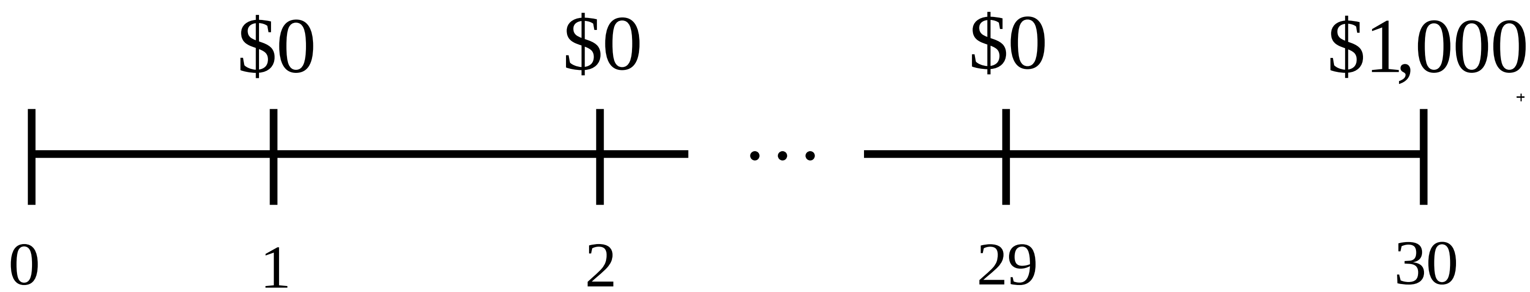


Present value of a pure discount bond at time 0:

$$PV = \frac{FV}{(1+r)^T}$$

Pure Discount Bond: Example

Find the value of a 30-year zero-coupon bond with a $1,000 par value and a YTM of 6%.



$$PV = \frac{FV}{(1+r)^T} = \frac{\$1{,}000}{(1.06)^{30}} = \$174.11$$

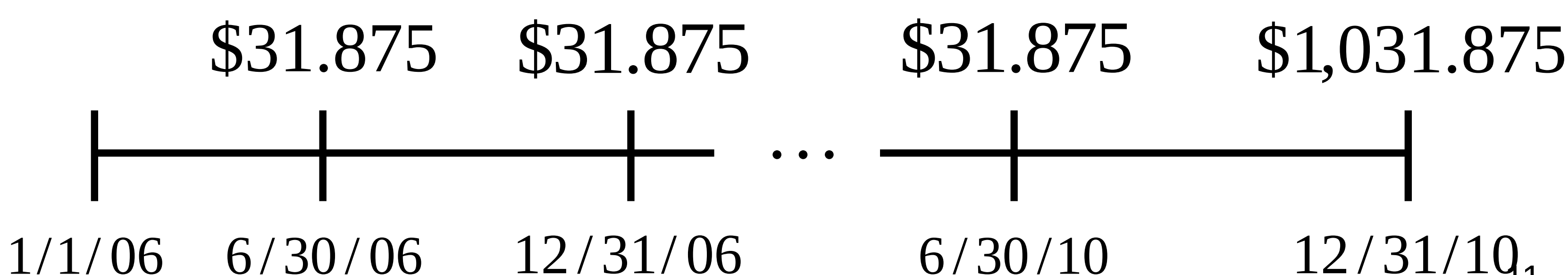
Level Coupon Bonds

- Make periodic coupon payments in addition to the maturity value
- The payments are equal in each period. Therefore, the bond is just a combination of an annuity and a terminal (maturity) value
- Coupon payments are typically semiannual
- Effective annual rate (EAR):

$$\left(1+\frac{r}{m}\right)^{m}-1$$

Level Coupon Bond: Example

- Consider a U.S. government bond with a 6 3/8% coupon, acquired in January 2006 that expires in December 2010.
  - The *Par Value* of the bond is $1,000.
  - *Coupon payments* are made semi-annually (June 30 and December 31 for this particular bond).
  - Since the *coupon rate* is 6 3/8%, the payment is $31.875.
  - On January 1, 2006 the size and timing of cash flows are:



Level Coupon Bond: Example

- On January 1, 2010, the required annual yield is 5%.

$$PV = \frac{\$31.875}{.05/2}\left[1 - \frac{1}{(1.025)^{10}}\right] + \frac{\$1{,}000}{(1.025)^{10}} = \$1{,}060.17$$

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Consols

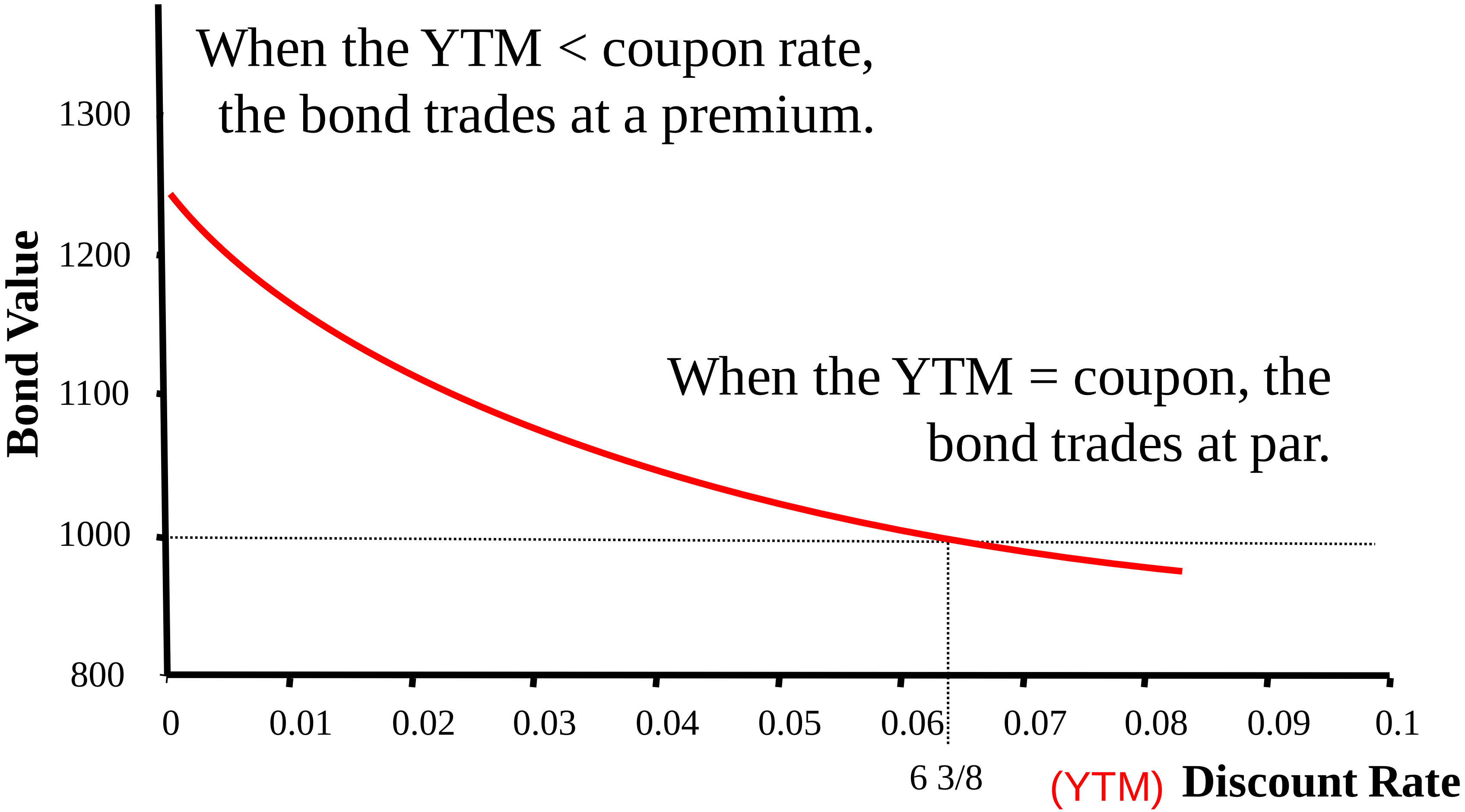
- Not all bonds have a final maturity
- British consols pay a set amount (i.e., coupon) every period forever
- These are examples of a perpetuity

$$PV = \frac{C}{R}$$

3. Key Bond Concepts

- 2 key characteristics:
  interest rate & maturity
- Bond prices and market interest rates move in opposite directions
  - When coupon rate = YTM, price = par value (par bond)
  - When coupon rate > YTM, price > par value (premium bond)
  - When coupon rate < YTM, price < par value (discount bond)

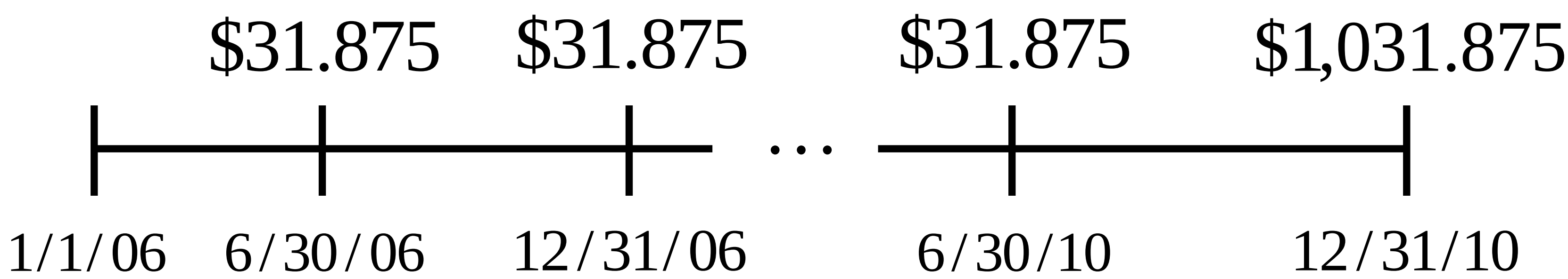
YTM and Bond Value



When the YTM > coupon rate, the bond trades at a discount.

Level Coupon Bond Example Revisited

- Using our previous example, now assume that the required yield is 11%.
- How does this change the bond's price?



$$PV = \frac{\$31.875}{.11/2}\left[1 - \frac{1}{(1.055)^{10}}\right] + \frac{\$1,000}{(1.055)^{10}} = \$825.69$$

Computing Yield to Maturity

- Yield to maturity is the rate implied by the current bond price
- Finding the YTM requires trial and error if you do not have a financial calculator and is similar to the process for finding the discount rate (*R)* with an annuity

YTM with Annual Coupons

- Consider a bond with a 10% annual coupon rate, 15 years to maturity, and a par value of £1,000. The current price is £928.09.
  - Will the yield be more or less than 10%?
  - Procedure of using a financial calculator:
  - N = 15; PV = -928.09; FV = 1,000; PMT = 100
  - CPT I/Y = 11%

YTM with Semiannual Coupons

- Suppose a bond with a 10% coupon rate and semiannual coupons has a face value of £1,000, 20 years to maturity, and is selling for £1,197.93.
  - Is the YTM more or less than 10%?
  - What is the semiannual coupon payment?
  - How many periods are there?
  - N = 40; PV = -1,197.93; PMT = 50; FV = 1,000; CPT I/Y = 4% (Is this the YTM?)
  - YTM = 4%*2 = 8%

4. The Present Value of Common Stocks

- The value of any asset is the present value of its expected future cash flows
- Stock ownership produces cash flows from:
  - Dividends
  - Capital Gains

The formula

$$P_0 = \frac{\text{Div}_1}{1+R} + \frac{P_1}{1+R}$$

$$P_0 = \frac{1}{1+R}\left[\text{Div}_1 + \left(\frac{\text{Div}_2 + P_2}{1+R}\right)\right]$$

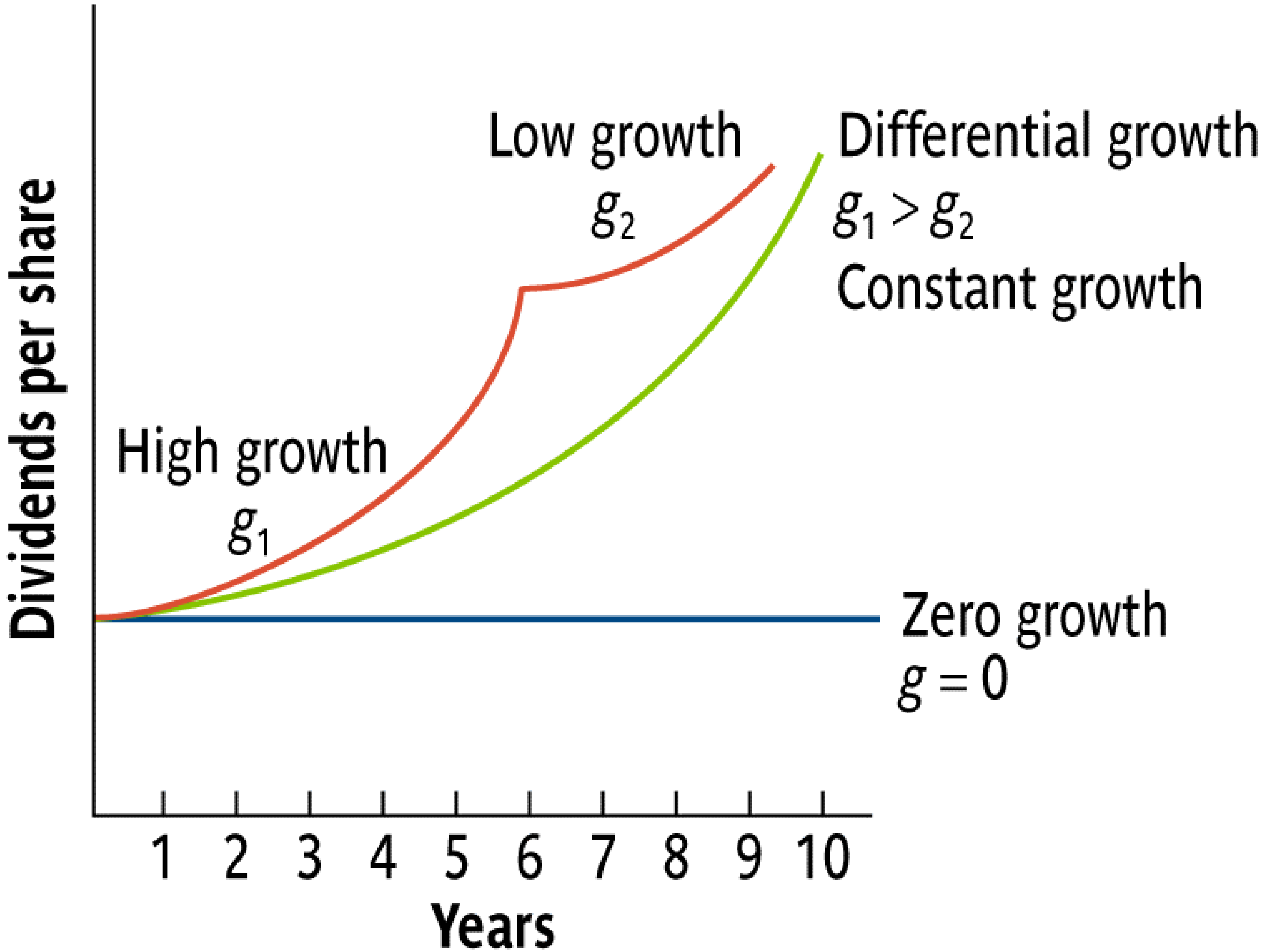
$$P_1 = \frac{\text{Div}_2}{1+R} + \frac{P_2}{1+R}$$

$$= \frac{\text{Div}_1}{1+R} + \frac{\text{Div}_2}{(1+R)^2} + \frac{P_2}{(1+R)^2}$$

$$P_0 = \frac{\text{Div}_1}{1+R} + \frac{\text{Div}_2}{(1+R)^2} + \frac{\text{Div}_3}{(1+R)^3} + \ldots = \sum_{t=1}^{\infty} \frac{\text{Div}_\text{t}}{(1+R)^t}$$

Equity valuation: three scenarios

- This approach to valuing stock is based on the *Dividend Growth Model*
- It has three variants depending on the assumptions (forecasts) we have about the growth of earnings (dividends):
  - Zero Growth
  - Constant Growth
  - Differential Growth


Dividends per share
Low growth
g_2
Differential growth
$g_1 > g_2$
Constant growth
High growth
g_1
Zero growth
$g = 0$
1 2 3 4 5 6 7 8 9 10
Years

Dividend growth models:

$$\text{Zero growth: } P_0 = \frac{\text{Div}_1}{R}$$

$$\text{Constant growth: } P_0 = \frac{\text{Div}_1}{R - g}$$

$$\text{Differential growth: } P_0 = \sum_{t=1}^{T} \frac{\text{Div}(1 + g_1)^t}{(1 + R)^t} + \frac{\frac{\text{Div}_{T+1}}{R - g_2}}{(1 + R)^T}$$

Case 1: Zero Growth

- Assume that dividends will remain at the same level forever

$$\text{Div}_1 = \text{Div}_2 = \text{Div}_3 = \cdots$$

- Since future cash flows are constant, the value of a zero growth stock is the present value of a perpetuity:

$$P_0 = \frac{\text{Div}_1}{(1+R)^1} + \frac{\text{Div}_2}{(1+R)^2} + \frac{\text{Div}_3}{(1+R)^3} + \cdots$$

$$P_0 = \frac{\text{Div}}{R}$$

Case 2: Constant Growth

Assume that dividends will grow at a constant rate, *g*, forever, *i.e.*,

$$\text{Div}_1 = \text{Div}_0(1+g)$$

$$\text{Div}_2 = \text{Div}_1(1+g) = \text{Div}_0(1+g)^2$$

$$\text{Div}_3 = \text{Div}_2(1+g) = \text{Div}_0(1+g)^3 \ldots$$

Since future cash flows grow at a constant rate forever, the value of a constant growth stock is the present value of a growing perpetuity:

$$P_0 = \frac{\text{Div}_1}{R-g}$$

Constant Growth Example

- Suppose ABC, Ltd., just paid a dividend of £.50. It is expected to increase its dividend by 2% per year. If the market requires a return of 15% on assets of this risk level, how much should the stock be selling for?

- P_0 = .50(1+.02) / (.15 - .02) = £3.92

Case 3: Differential Growth

- Assume that dividends will grow at different rates in the foreseeable future and then will grow at a constant rate thereafter
- To value a Differential Growth Stock, we need to:
  - Estimate future dividends in the foreseeable future
  - Estimate the future stock price when the stock becomes a Constant Growth Stock (case 2)
  - Compute the total present value of the estimated future dividends and future stock price at the appropriate discount rate

Case 3: Differential Growth

- Assume that dividends will grow at rate g_1 for N years and grow at rate g_2 thereafter.

$$\text{Div}_1 = \text{Div}_0(1+g_1)$$

$$\text{Div}_2 = \text{Div}_1(1+g_1) = \text{Div}_0(1+g_1)^2$$

$$\cdots$$

$$\text{Div}_N = \text{Div}_{N-1}(1+g_1) = \text{Div}_0(1+g_1)^N$$

$$\text{Div}_{N+1} = \text{Div}_N(1+g_2) = \text{Div}_0(1+g_1)^N(1+g_2)$$

$$\cdots$$

Case 3: Differential Growth

Dividends will grow at rate g_1 for N years and grow at rate g_2 thereafter

$$\text{Div}_0(1+g_1) \qquad \text{Div}_0(1+g_1)^2$$

$0 \qquad 1 \qquad 2 \qquad \cdots$

$$\text{Div}_0(1+g_1)^N$$

$$\text{Div}_N(1+g_2) = \text{Div}_0(1+g_1)^N(1+g_2)$$

$\cdots \qquad N \qquad N+1 \qquad \cdots$

Case 3: Differential Growth

We can value this as the sum of:

- an N-year annuity growing at rate g_1

$$P_A = \frac{Div_1}{R-g_1}\left[1-\frac{(1+g_1)^N}{(1+R)^N}\right]$$

- plus the discounted value of a perpetuity growing at rate g_2 that starts in year N+1

$$P_B = \frac{\left(\dfrac{\text{Div}_{\text{N+1}}}{R-g_2}\right)}{(1+R)^N}$$

Case 3: Differential Growth

Consolidating gives:

$$P = \frac{Div_1}{R - g_1}\left[1 - \frac{(1+g_1)^N}{(1+R)^N}\right] + \frac{\frac{Div_{N+1}}{R - g_2}}{(1+R)^N}$$

Or, we can "cash flow" it out.

A Differential Growth Example

A common stock just paid a dividend of £2. The dividend is expected to grow at 8% for 3 years, then it will grow at 4% in perpetuity.

What is the stock worth if the discount rate is 12%?

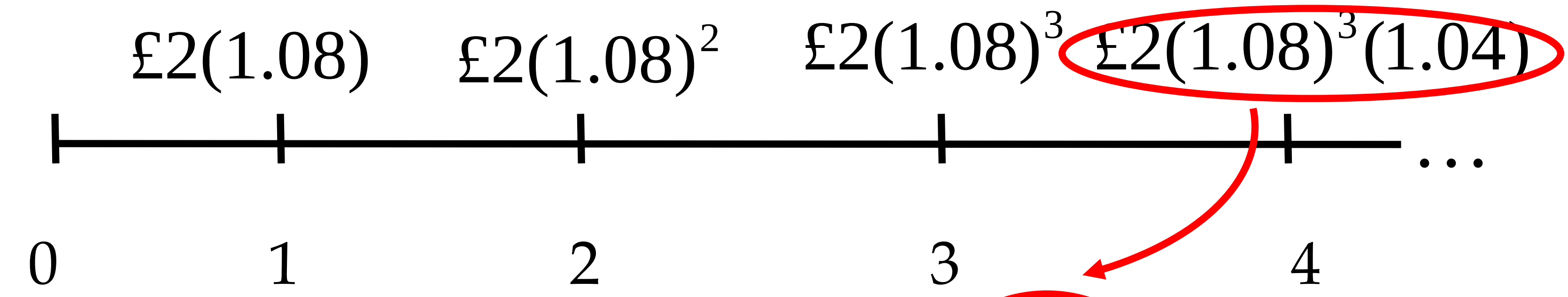
With the Formula

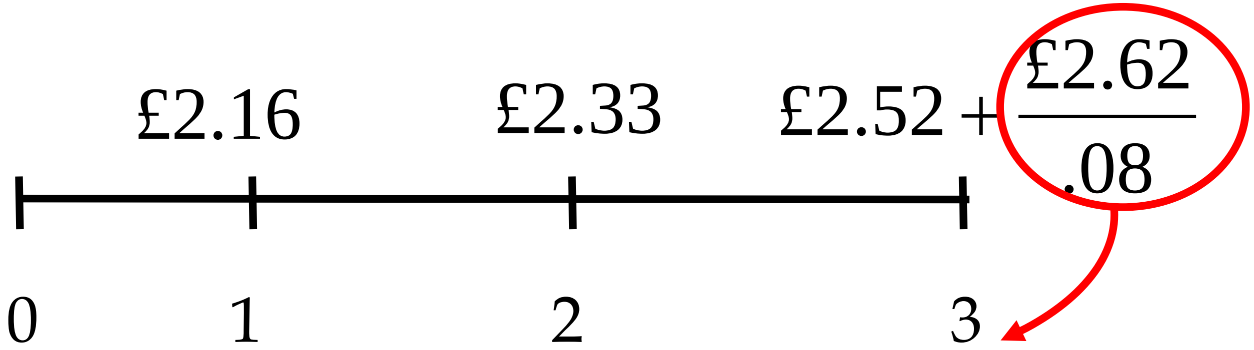
$$P = \frac{£2 \times (1.08)}{.12 - .08}\left[1 - \frac{(1.08)^3}{(1.12)^3}\right] + \frac{\left(\dfrac{£2(1.08)^3(1.04)}{.12 - .04}\right)}{(1.12)^3}$$

$$P = £54 \times [1 - .8966] + \frac{(£32.75)}{(1.12)^3}$$

$$P = £5.58 + £23.31 \qquad P = £28.89$$

With Cash Flows





The constant growth phase beginning in year 4 can be valued as a growing perpetuity *at time 3*.

$$P_0 = \frac{£2.16}{1.12} + \frac{£2.33}{(1.12)^2} + \frac{£2.52 + £32.75}{(1.12)^3} = £28.89$$

$$P_3 = \frac{£2.62}{.08} = £32.75$$

5. Estimates of Parameters

- Dividend Growth Model
- The value of a firm depends upon the rate of growth of earnings, g, and its discount rate, R
- 2 questions
  - Where does g come from?
  - Where does R come from?

Where Does g Come From?

$$\text{Earnings next year} = \text{Earnings this year} + \underbrace{\text{Retained earnings this year} \times \text{Return on retained earnings}}_{\text{Increase in earnings}}$$

$$\frac{\text{Earnings next year}}{\text{Earnings this year}} = \frac{\text{Earnings this year}}{\text{Earnings this year}} + \left(\frac{\text{Retained earnings this year}}{\text{Earnings this year}}\right) \times \text{Return on retained earnings}$$

$1 + g = 1 +$ (Retention ratio × Return on retained earnings)

$g =$ Retention ratio × Return on retained earnings

Example: earnings growth

- Pagemaster plc just reported earnings of £2 million. It plans to retain 40 percent of its earnings. The historical return on equity (ROE) has been 16 percent, a figure that is expected to continue into the future.
- How much will earnings grow over the coming year?

$$g = \text{Retention ratio} \times \text{ROE}$$

$$g = .4 \times .16 = .064$$

Where does *R* come from?

- The discount rate can be broken into two parts.
  - The dividend yield
  - The growth rate (in dividends)

- In practice, there is a great deal of estimation error involved in estimating *R*.

Using the DGM to Find R

- Start with the DGM:

$$P_0 = \frac{D_0(1+g)}{R - g} = \frac{D_1}{R - g}$$

Rearrange and solve for R:

$$R = \frac{D_0(1+g)}{P_0} + g = \frac{D_1}{P_0} + g$$

6. Problems with the dividend-based models for valuing stock

- Problem: many (listed or not) companies do not pay dividends…
- Mainstream corporate finance textbook dodge the issue by saying that we (investors and analysts) should expect that the company will **eventually** pay dividends **in the future**
- Even if that is true, it casts a long shadow on all calculations we just made …
- … Which means that the market value of stock really ultimately depends on what others think the value should be