Simultaneous Equations

- $y_1 = \alpha_1 y_2 + \beta_1 z_1 + u_1$
- $y_2 = \alpha_2 y_1 + \beta_2 z_2 + u_2$

Simultaneity

- Simultaneity is a specific type of endogeneity problem in which the explanatory variable is jointly determined with the dependent variable
- As with other types of endogeneity, IV estimation can solve the problem
- Some special issues to consider with simultaneous equations models (SEM)

Supply and Demand Example

- Start with an equation you'd like to estimate, say a labor supply function
- $h_s = \alpha_1 w + \beta_1 z + u_1$, where
- w is the wage and z is a supply shifter
- Call this a structural equation – it's derived from economic theory and has a causal interpretation where w directly affects h_s

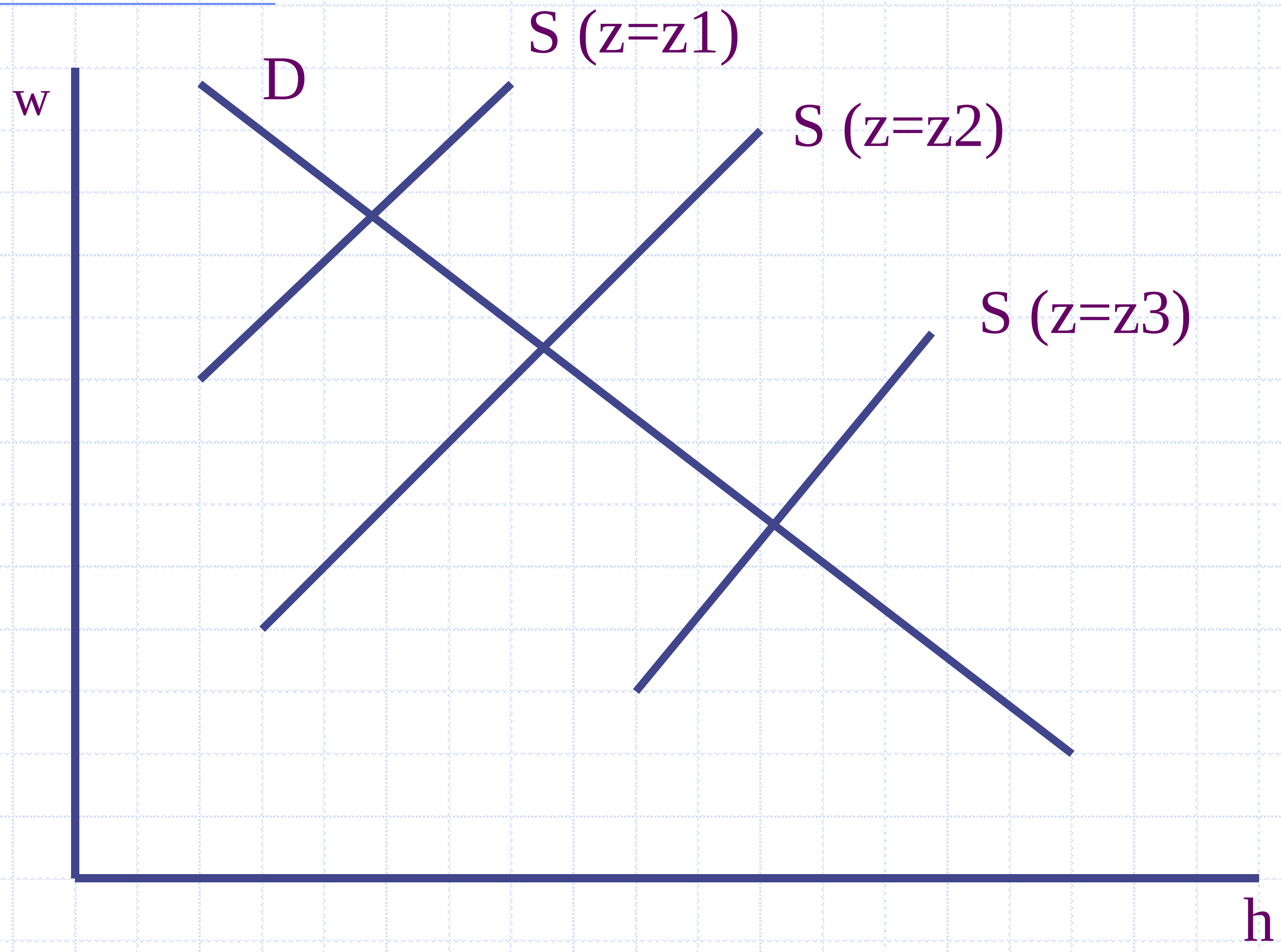
Example (cont)

- Problem that can't just regress observed hours on wage, since observed hours are determined by the equilibrium of supply and demand
- Consider a second structural equation, in this case the labor demand function
- $h_d = \alpha_2 w + u_2$
- So hours are determined by a SEM

Example (cont)

- Both *h* and *w* are endogenous because they are both determined by the equilibrium of supply and demand
- *z* is exogenous, and it's the availability of this exogenous supply shifter that allows us to identify the structural demand equation
- With no observed demand shifters, supply is not identified and cannot be estimated

Identification of Demand Equation



Using IV to Estimate Demand

- So, we can estimate the structural demand equation, using z as an instrument for w
- First stage equation is $w = \pi_0 + \pi_1 z + v_2$
- Second stage equation is $h = \alpha_2 \hat{w} + u_2$
- Thus, 2SLS provides a consistent estimator of α_2, the slope of the demand curve
- We cannot estimate α_1, the slope of the supply curve

The General SEM

- Suppose you want to estimate the structural equation: $y_1 = \alpha_1 y_2 + \beta_1 z_1 + u_1$
- where, $y_2 = \alpha_2 y_1 + \beta_2 z_2 + u_2$
- Thus, $y_2 = \alpha_2(\alpha_1 y_2 + \beta_1 z_1 + u_1) + \beta_2 z_2 + u_2$
- So, $(1 - \alpha_2 \alpha_1) y_2 = \alpha_2 \beta_1 z_1 + \beta_2 z_2 + \alpha_2 u_1 + u_2$, which can be rewritten as
- $y_2 = \pi_1 z_1 + \pi_2 z_2 + v_2$

The General SEM (continued)

- By substituting this reduced form in for y_2, we can see that since v_2 is a linear function of u_1, y_2 is correlated with the error term and α_1 is biased – call it simultaneity bias
- The sign of the bias is complicated, but can use the simple regression as a rule of thumb
- In the simple regression case, the bias is the same sign as $\alpha_2/(1 - \alpha_2\alpha_1)$

Identification of General SEM

- Let z_1 be all the exogenous variables in the first equation, and z_2 be all the exogenous variables in the second equation
- It's okay for there to be overlap in z_1 and z_2
- To identify equation 1, there must be some variables in z_2 that are not in z_1
- To identify equation 2, there must be some variables in z_1 that are not in z_2

Rank and Order Conditions

- We refer to this as the rank condition
- Note that the exogenous variable excluded from the first equation must have a non-zero coefficient in the second equation for the rank condition to hold
- Note that the order condition clearly holds if the rank condition does – there will be an exogenous variable for the endogenous one

Estimation of the General SEM

- Estimation of SEM is straightforward
- The instruments for 2SLS are the exogenous variables from both equations
- Can extend the idea to systems with more than 2 equations
- For a given identified equation, the instruments are all of the exogenous variables in the whole system